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LOW-COMPLEXITY ROBUST ADAPTIVE BEAMFORMING ALGORITHMS EXPLOITING SHRINKAGE FOR MISMATCH ESTIMATION

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Outline

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Introduction

- Conventional adaptive beamforming algorithms are sensitive to steering vector mismatches (imperfect array calibration, fading, scattering etc).

- Robust Adaptive Beamforming (RAB) methods have been developed in the last years to mitigate the effects of mismatches , which include:
  - Diagonal loading.
  - Worst-case optimization.
  - Subspace techniques.

- However, robust algorithms have performance limitations and are costly (cubic or greater costs) especially for large sensor arrays.

H. Cox, R. M. Zeskind, and M. M. Owen, “Robust adaptive beamforming,” IEEE Trans. Acoust., Speech, Signal Processing, vol. ASSP-35, pp. 1365–1376, Oct. 1987.
S. A. Vorobyov, A. B. Gershman and Z. Luo, “Robust Adaptive Beamforming using Worst-Case Performance Optimization: A Solution to the Signal Mismatch Problem,” IEEE Transactions on Signal Processing, Vol. 51, No. 4, pp 313-324, Feb 2003.
D. D. Feldman and L. J. Griffiths, “A projection approach for robust adaptive beamforming,” IEEE Trans. Signal Processing, vol. 42, pp. 867–876, Apr. 1994

Contributions

- Proposed LOCSME-CG robust beamforming method:
  - exploits prior knowledge that the steering vector mismatch resided in a known angular sector.
  - relies on the cross-correlation between the received data and the output of the beamformer.
  - mismatch is preliminary computed using the projection of a shrinkage-estimated cross-correlation vector onto an eigen subspace.
  - mismatch is further updated with CG based recursions.
  - weight vector is also obtained with CG based recursions in a reduced complexity.

- The proposed LOCSME-CG approach is cost effective and is compared with previously reported algorithms.

System Model

- We assume narrowband signals in the far field with zero mean, which can be treated as point sources producing plane waves at the sensor array.

- Signal model:

$$\boldsymbol{x}(i) = \boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{s}(i) + \boldsymbol{n}(i).$$

where $\boldsymbol{x}(i) \in \mathbb{C}^{M\times 1}$ is the received signal at the sensor arry.

$\boldsymbol{s}(i) \in \mathbb{C}^{K\times 1}$ is the signal sources vector including the desired signal and K interferers.

$\boldsymbol{A}(\boldsymbol{\theta}) = [\boldsymbol{a}(\theta_1), \dots, \boldsymbol{a}(\theta_K)] = [\boldsymbol{a}_1, \dots, \boldsymbol{a}_K] \in \mathbb{C}^{M\times K}$.

$\boldsymbol{\theta} = [\theta_1, \dots, \theta_K]^T \in \mathbb{R}^{K\times 1}$ contains the directions of arrival.

$\boldsymbol{n}(i)$ complex circular Gaussian noise with zero mean and ${\sigma_n}^2$.

- Beamforming:

$$\mathrm{y(i)} = \boldsymbol{w}^H(i)\boldsymbol{x}(i).$$

where $\boldsymbol{w}(i)$ is the beamforming weight vector.

Problem Statement

- The output SINR is determined by

$$SINR(i) = \frac{\sigma_1{}^2(i)|\boldsymbol{w}^H(i)\boldsymbol{a}_1(i)|^2}{\boldsymbol{w}^H(i)\boldsymbol{R}_{i+n}(i)\boldsymbol{w}(i)}$$

where $\sigma_1{}^2(i)$ is the desired signal power.

$\boldsymbol{R}_{i+n}(i)$ is the interference-plus-noise covariance matrix.

- Quantities that need to be estimated: $\sigma_1{}^2(i)$, $\boldsymbol{a}_1(i)$, $\boldsymbol{R}_{i+n}(i)$ and $\boldsymbol{w}(i)$.

Key problem:

- To design a cost-efficient RAB algorithm that can precisely and efficiently estimate the steering vector mismatch and update the beamforming weight vector, from an implementation point of view.

Proposed LOCSME-CG algorithm: Starting with the cross-correlation vector

- Prior knowledge: we assume that $\boldsymbol{a}_1(i)$ is located inside a pre-known angular sector and exploit the cross-correlation between $\boldsymbol{x}(i)$ and y(i) as

$$\boldsymbol{d}(i) = E[\boldsymbol{x}(i)y^*(i)] = E[\big(\boldsymbol{A}\boldsymbol{s}(i) + \boldsymbol{n}(i)\big)\big(\boldsymbol{A}\boldsymbol{s}(i) + \boldsymbol{n}(i)\big)^H \boldsymbol{w}(i)]$$

where $E[.]$ denotes the expectation operator.

- We assume the desired signal is independent from the interferers and noise:

$$\boldsymbol{d}(i) = E[(\boldsymbol{A}\boldsymbol{s}(i)\boldsymbol{s}^H(i)\boldsymbol{A}^H)\boldsymbol{w}(i) + \boldsymbol{n}(i)\boldsymbol{n}^H(i)\boldsymbol{w}(i)]$$

- We also assume $|\boldsymbol{a}_m{}^H(i)\boldsymbol{w}(i)| \ll |\boldsymbol{a}_1{}^H(i)\boldsymbol{w}(i)|$ for $m = 2,3,\ldots,K$:

$$\boldsymbol{d}(i) = E[\sigma_1{}^2(i)\boldsymbol{a}_1{}^H(i)\boldsymbol{w}(i)\boldsymbol{a}_1(i) + \boldsymbol{n}(i)\boldsymbol{n}^H(i)\boldsymbol{w}(i)]$$

which depends on the desired signal and the noise.

- The vector $\boldsymbol{d}(i)$ can be estimated with the sample cross-correlation vector as

$$\widehat{\boldsymbol{d}}(i) = \frac{1}{i}\sum_{j=1}^{i} \boldsymbol{x}(j)y^*(j)$$

Proposed LOCSME-CG algorithm: shrinkage of the cross-correlation vector

- A linear shrinkage method is adopted to process a shrinkage estimation for vector $\widehat{\boldsymbol{d}}(i)$:

$$\hat{v}(i) = \sum \widehat{\boldsymbol{d}}(i)\,/M,$$
$$\widehat{\boldsymbol{d}}(i) = \hat{\rho}(i)\hat{v}(i) + \big(1 - \hat{\rho}(i)\big)\widehat{\boldsymbol{d}}(i-1)$$

- where $\hat{\rho}(i)$ is the shrinkage coefficient within 0 to 1 and can be obtained by minimizing the MSE of $E\left[\left\|\widehat{\boldsymbol{d}}(i) - \widehat{\boldsymbol{d}}(i-1)\right\|^2\right]$, resulting in

$$\hat{\rho}(i) = \frac{\left(1 - \frac{2}{M}\right)\widehat{\boldsymbol{d}}^H(i-1)\widehat{\boldsymbol{d}}(i-2) + \sum \widehat{\boldsymbol{d}}(i-1)\sum \widehat{\boldsymbol{d}}^*(i-1)}{\left(i - \frac{2}{M}\right)\widehat{\boldsymbol{d}}^H(i-1)\widehat{\boldsymbol{d}}(i-2) + (1 - \frac{i}{M})\sum \widehat{\boldsymbol{d}}(i-1)\sum \widehat{\boldsymbol{d}}^*(i-1)}$$

Proposed LOCSME-CG algorithm: subspace projection

- With a presumed angular sector $[\theta_1 - \theta_e, \theta_1 + \theta_e]$, we can define a projection subspace as

$$\boldsymbol{P} = [\boldsymbol{c}_1, \ldots, \boldsymbol{c}_p][\boldsymbol{c}_1, \ldots, \boldsymbol{c}_p]^H$$

where $\boldsymbol{c}_1, \ldots, \boldsymbol{c}_p$ are the first p principal eigenvectors of $\boldsymbol{C}$ computed by

$$\boldsymbol{C} = \int_{\theta_1-\theta_e}^{\theta_1+\theta_e} \boldsymbol{a}(\theta)\boldsymbol{a}^H(\theta)d\theta$$

- The steering vector is then obtained by projecting $\widehat{\boldsymbol{d}}(i)$ onto the subspace

$$\widehat{\boldsymbol{a}}_1(i) = \frac{\boldsymbol{P}(i)\widehat{\boldsymbol{d}}(i)}{\left\|\boldsymbol{P}(i)\widehat{\boldsymbol{d}}(i)\right\|}$$

Proposed LOCSME-CG algorithm: Desired signal power estimation

- Suitable for orthogonal or quasi-orthogonal signals.

- The signal $\boldsymbol{x}(i)$ can be expanded as

$$\boldsymbol{x}(i) = \hat{\boldsymbol{a}}_1(i)s_1(i) + \sum_{k=2}^{K} \boldsymbol{a}_k(i)s_k(i) + \boldsymbol{n}(i)$$

where $\hat{\boldsymbol{a}}_1(i)$ denotes the estimate of $\boldsymbol{a}_1(i)$.

- Pre-multiply it by $\hat{\boldsymbol{a}}_1{}^H(i)$ on the left assuming the signals are orthogonal:

$$\hat{\boldsymbol{a}}_1{}^H(i)\boldsymbol{x}(i) = \hat{\boldsymbol{a}}_1{}^H(i)\hat{\boldsymbol{a}}_1(i)s_1(i) + \hat{\boldsymbol{a}}_1{}^H(i)\boldsymbol{n}(i)$$

- Taking the expectation of $|\hat{\boldsymbol{a}}_1{}^H(i)\boldsymbol{x}(i)|^2$, simplifying it using $E[\boldsymbol{n}(i)\boldsymbol{n}^H(i)] = \sigma_n{}^2\boldsymbol{I}_M$ and replacing $|s_1(i)|^2$ by $\hat{\sigma}_1{}^2(i)$, the desired signal power is estimated as

$$\hat{\sigma}_1{}^2(i) = \frac{|\hat{\boldsymbol{a}}_1{}^H(i)\boldsymbol{x}(i)|^2 - |\hat{\boldsymbol{a}}_1{}^H(i)\hat{\boldsymbol{a}}_1(i)|\sigma_n{}^2(i)}{|\hat{\boldsymbol{a}}_1{}^H(i)\hat{\boldsymbol{a}}_1(i)|^2}$$

Proposed LOCSME-CG algorithm: CG based recursions

- For both the desired signal steering vector and the weight vector we have

$$\hat{\mathbf{a}}_1(i) = \hat{\mathbf{a}}_1(i-1) + \alpha_{\hat{\mathbf{a}}_1}(i)\mathbf{p}_{\hat{\mathbf{a}}_1}(i)$$

$$\mathbf{v}(i) = \mathbf{v}(i-1) + \alpha_{\mathbf{v}}(i)\mathbf{p}_{\mathbf{v}}(i)$$

where a degenerated scheme is employed to guarantee convergence:

$$0 \leq \mathbf{p}_{\hat{\mathbf{a}}_1}^H(i)\mathbf{g}_{\hat{\mathbf{a}}_1}(i) \leq 0.5\mathbf{p}_{\hat{\mathbf{a}}_1}^H(i)\mathbf{g}_{\hat{\mathbf{a}}_1}(i-1)$$

$$0 \leq \mathbf{p}_{\mathbf{v}}^H(i)\mathbf{g}_{\mathbf{v}}(i) \leq 0.5\mathbf{p}_{\mathbf{v}}^H(i)\mathbf{g}_{\mathbf{v}}(i-1)$$

where $\mathbf{g}_{\hat{\mathbf{a}}_1}(i)$ and $\mathbf{g}_{\mathbf{v}}(i)$ are the negative gradient vectors, which are updated using a forgetting factor as

$$\begin{aligned}\mathbf{g}_{\hat{\mathbf{a}}_1}(i) &= (1-\lambda)\mathbf{v}(i) + \lambda\mathbf{g}_{\hat{\mathbf{a}}_1}(i-1) \\ &+ \hat{\sigma}_1^2(i)\alpha_{\hat{\mathbf{a}}_1}(i)\mathbf{v}(i)\mathbf{v}^H(i)\mathbf{p}_{\hat{\mathbf{a}}_1}(i) - \mathbf{x}(i)\mathbf{x}^H(i)\hat{\mathbf{a}}_1(i)\end{aligned}$$

$$\begin{aligned}\mathbf{g}_{\mathbf{v}}(i) &= (1-\lambda)\hat{\mathbf{a}}_1(i) + \lambda\mathbf{g}_{\mathbf{v}}(i-1) - \alpha_{\mathbf{v}}(i)(\mathbf{R}(i) \\ &- \hat{\sigma}_1^2(i)\hat{\mathbf{a}}_1(i)\hat{\mathbf{a}}_1^H(i))\mathbf{p}_{\mathbf{v}}(i) - \mathbf{x}(i)\mathbf{x}^H(i)\mathbf{v}(i-1)\end{aligned}$$

Proposed LOCSME-CG algorithm: CG based recursions

By pre-multiply the above two equations by $\mathbf{p}_{\hat{\mathbf{a}}_1}^H(i)$ and $\mathbf{p}_{\mathbf{v}}^H(i)$ respectively, taking the expectations and simplifying we obtain

$$\alpha_{\hat{\mathbf{a}}_1}(i) = [\lambda(\mathbf{p}_{\hat{\mathbf{a}}_1}^H(i)\mathbf{v}(i) - \mathbf{p}_{\hat{\mathbf{a}}_1}^H(i)\mathbf{g}_{\hat{\mathbf{a}}_1}(i-1)) - \mathbf{p}_{\hat{\mathbf{a}}_1}^H(i)\mathbf{v}(i)$$
$$+\mathbf{p}_{\hat{\mathbf{a}}_1}^H(i)\mathbf{x}(i)\mathbf{x}^H(i)\hat{\mathbf{a}}_1(i) + \eta_{\hat{\mathbf{a}}_1}\mathbf{p}_{\hat{\mathbf{a}}_1}^H(i)\mathbf{g}_{\hat{\mathbf{a}}_1}(i-1)]$$
$$/[\hat{\sigma}_1^2(i)\mathbf{p}_{\hat{\mathbf{a}}_1}^H(i)\mathbf{v}(i)\mathbf{v}^H(i)\mathbf{p}_{\hat{\mathbf{a}}_1}(i)]$$

$$\alpha_{\mathbf{v}}(i) = \frac{\bar{\lambda}(\mathbf{p}_{\mathbf{v}}^H(i)\mathbf{g}_{\mathbf{v}}(i-1) - \mathbf{p}_{\mathbf{v}}^H(i)\hat{\mathbf{a}}_1(i)) - \eta_{\mathbf{v}}\mathbf{p}_{\mathbf{v}}^H(i)\mathbf{g}_{\mathbf{v}}(i-1)}{\mathbf{p}_{\mathbf{v}}^H(i)(\hat{\mathbf{R}}(i) - \hat{\sigma}_1^2(i)\hat{\mathbf{a}}_1(i)\hat{\mathbf{a}}_1^H(i))\mathbf{p}_{\mathbf{v}}(i)}$$

Then $\mathbf{p}_{\hat{\mathbf{a}}_1}^H(i)$ and $\mathbf{p}_{\mathbf{v}}^H(i)$ are updated as

$$\mathbf{p}_{\hat{\mathbf{a}}_1}(i+1) = \mathbf{g}_{\hat{\mathbf{a}}_1}(i) + \beta_{\hat{\mathbf{a}}_1}(i)\mathbf{p}_{\hat{\mathbf{a}}_1}(i)$$
$$\mathbf{p}_{\mathbf{v}}(i+1) = \mathbf{g}_{\mathbf{v}}(i) + \beta_{\mathbf{v}}(i)\mathbf{p}_{\mathbf{v}}(i)$$

Proposed LOCSME-CG algorithm: CG based recursions

where

$$\beta_{\hat{\mathbf{a}}_1}(i) = \frac{[\mathbf{g}_{\hat{\mathbf{a}}_1}(i) - \mathbf{g}_{\hat{\mathbf{a}}_1}(i-1)]^H \mathbf{g}_{\hat{\mathbf{a}}_1}(i)}{\mathbf{g}_{\hat{\mathbf{a}}_1}^H(i-1)\mathbf{g}_{\hat{\mathbf{a}}_1}(i-1)}$$

$$\beta_{\mathbf{v}}(i) = \frac{[\mathbf{g}_{\mathbf{v}}(i) - \mathbf{g}_{\mathbf{v}}(i-1)]^H \mathbf{g}_{\mathbf{v}}(i)}{\mathbf{g}_{\mathbf{v}}^H(i-1)\mathbf{g}_{\mathbf{v}}(i-1)}$$

Finally the beamformer weight vector is computed with

$$\mathbf{w}(i) = \frac{\mathbf{v}(i)}{\hat{\mathbf{a}}_1^H(i)\mathbf{v}(i)}$$

- Note that the constant parameters λ and η need to be adjusted to achieve the best performance
- Only one iteration is performed per snapshot

Computational Complexity

RAB algorithms	Number of flops
LOCSME (the batch method) – A)	$4M^3 + 3M^2 + 20M$
LOCSME-SG -D)	$15M^2 + 30M$
Covariance matrix shrinkage – B)	$M^{3.5} + 7M^3 + 5M^2 + 3M$
LCWC – C)	$100M^2 + 350M$
LOCSME-CG	$13M^2 + 77M$

A) H. Ruan and R. C. de Lamare, "Robust Adaptive Beamforming Using a Low-Complexity Shrinkage-Based Mismatch Estimation Algorithm," IEEE Sig. Proc. Letters., Vol. 21, No. 1, pp 60-64, 2013.

B) Y. Gu and A. Leshem, "Robust Adaptive Beamforming Based on Jointly Estimating Covariance Matrix and Steering Vector," Proc. IEEE International Conference on Acoustics Speech and Signal Processing, pp 2640-2643, 2011.

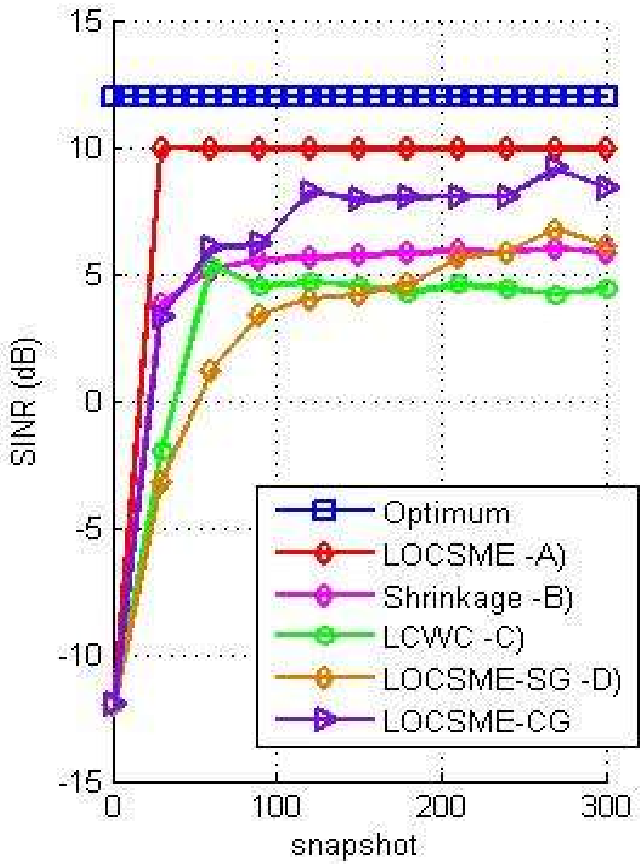
C) A. Elnashar, "Efficient implementation of robust adaptive beamforming based on worst-case performance optimization," IET Signal Process., Vol. 2, No. 4, pp. 381-393, Dec 2008.

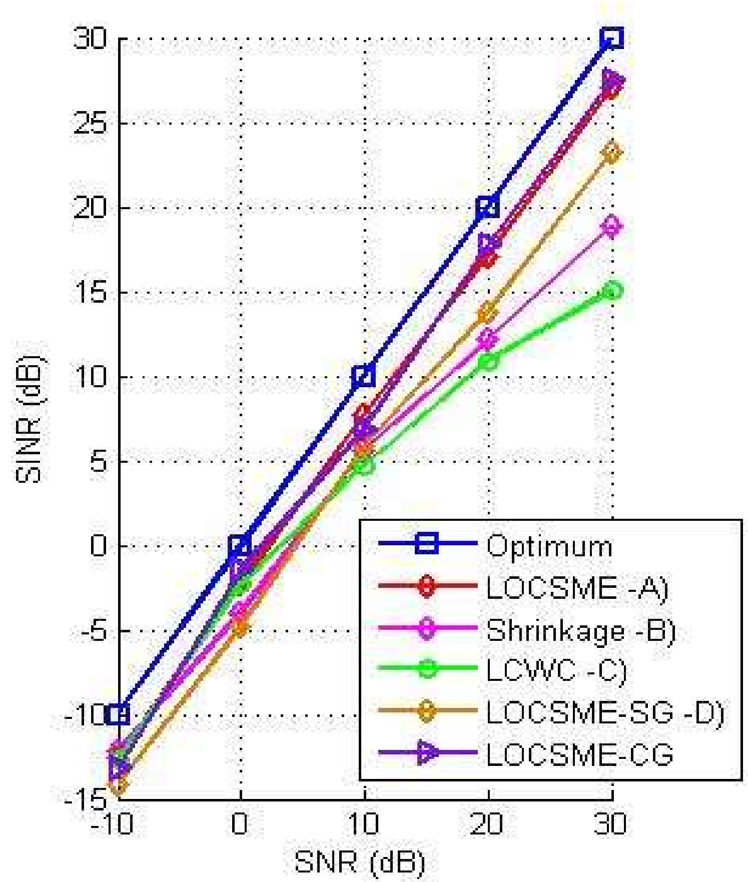
D) H. Ruan and R. C. de Lamare, "Low-Complexity Robust Adaptive Beamforming Based on Shrinkage and Cross-Correlation," 19th International ITG Workshop on Smart Antennas, pp 1-5, March 2015.

Simulation setup

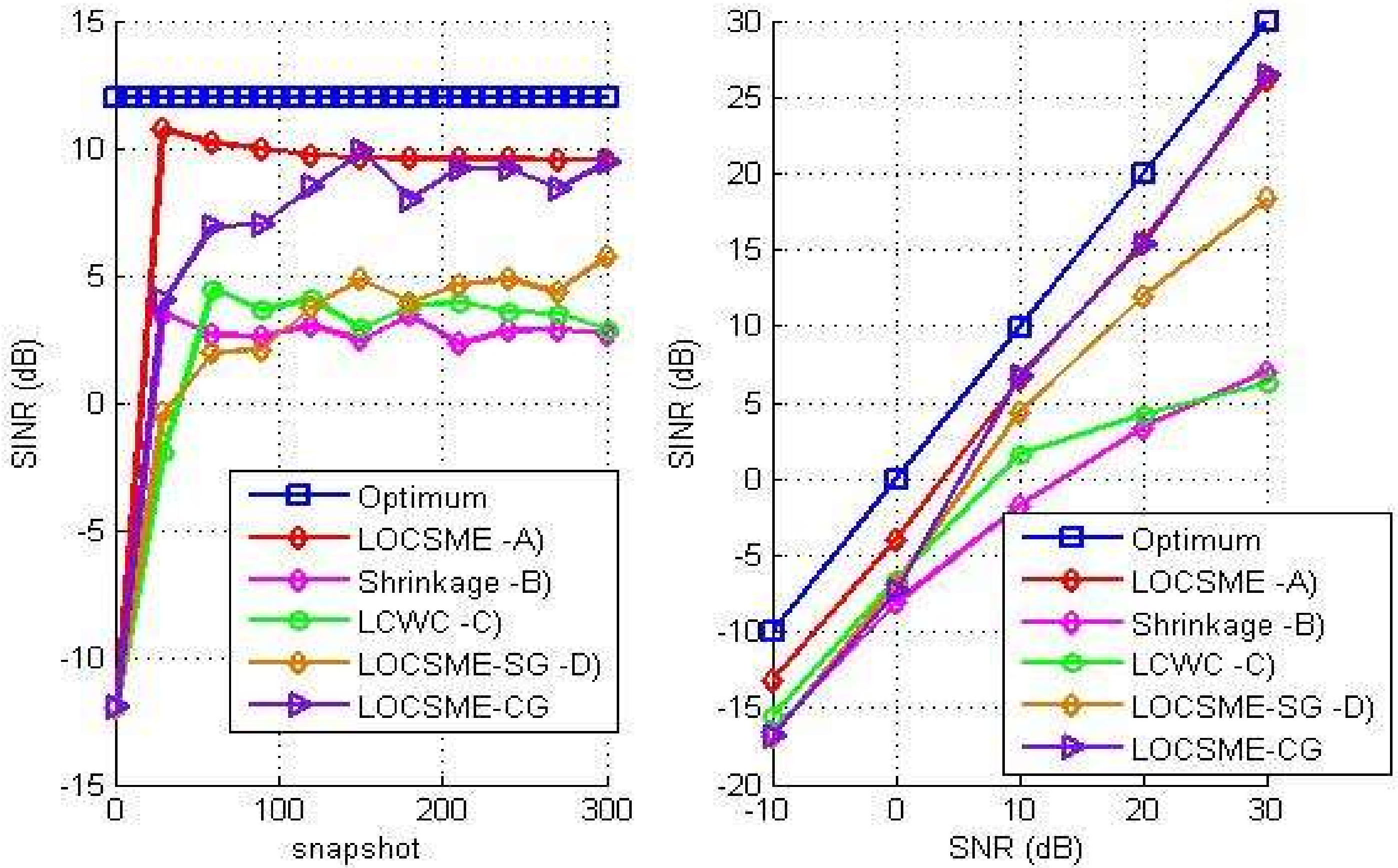
- We consider a scenario with 3 signals (2 interferers).
- Source angles are $\theta_1 = 10°, \theta_2 = 30°, \theta_3 = 50°$ and the presumed angular sector is set as $[5°, 15°]$.
- A Uniform Linear Array (ULA) of $M = 12$ omnidirectional sensors and half-wavelength spacing is used.
- Coherent and incoherent local scattering scenarios are considered for modeling the mismatch.
- Approaches used in the comparisons:
  - Optimum SINR
  - LOCSME – A)
  - Covariance matrix shrinkage – B)
  - LCWC –C)
  - LOCSME-SG –D)

Simulation results: coherent local scattering





Simulation results: incoherent local scattering



Conclusions

- We have developed an efficient approach for steering vector mismatch estimation:
  - Exploitation of the cross-correlation vector and its shrinkage effects
  - Assumption of known angular sector
  - CG based recursions

- We have compared the proposed method to existing methods and shown its advantages on the following aspects:
  - Lower computational complexity
  - Superior SINR performance

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