

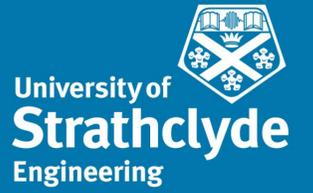
Cyclic-by-Row Approximation of Iterative Polynomial EVD Algorithms

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Background

- ▶ Polynomial Matrix: a matrix where each element is a Laurent polynomial giving it a 3D structure.
 - ▶ Parahermitian matrix: extends the idea of Hermitian matrices to polynomial matrices, in addition to symmetry across the diagonal there is also a time reversal about the zero lag $\tilde{R}(z) = R^H(z^{-1})$.
- Parahermitian polynomial matrices occur in scenarios such as broadband sensor array problems, e.g. [1], which require time delays as opposed to phase shifts.

The polynomial EVD (PEVD) is generalised in [2] as

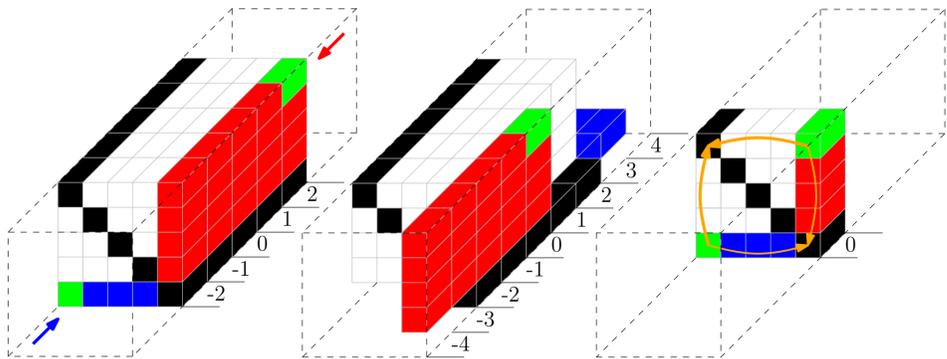
$$R(z) \approx H(z)\Gamma(z)\tilde{H}(z), \quad (1)$$

where $H(z)$ is paraunitary i.e. $H(z)\tilde{H}(z) = I$ and $\Gamma(z)$ is diagonal and spectrally majorised [3]. Iterative PEVD algorithms such as the second order sequential best rotation algorithm (SBR2) [2] can be used to approximate a solution to (1).

Iterative PEVD Algorithms

Iterative PEVD algorithms consist of three major steps:

1. Determine the elements to be shifted onto the zero lag
2. Shift the appropriate row and column onto the zero lag
3. Transfer energy from the zero lag onto the diagonal



1. $\{k^{(i)}, \tau^{(i)}\} = \arg \max_{k, \tau} \|\tilde{s}_k^{(i-1)}[\tau]\|_\infty$
2. $S^{(i)'}(z) = \tilde{\Lambda}^{(i)}(z)S^{(i-1)}(z)\Lambda^{(i)}(z)$
3. $S^{(i)}(z) = Q^{(i)H}S^{(i)'}(z)Q^{(i)}$

- ▶ SBR2 step 3 done using a Jacobi transformation applied to all lags.
- ▶ Sequential matrix diagonalisation (SMD) [4] algorithms use a full EVD of the zero lag (applied to all lags) for step 3.

The product of these steps over I iterations provides the paraunitary matrix

$$H(z) = \prod_{i=1}^I \Lambda^{(i)}(z)Q^{(i)}. \quad (2)$$

Cyclic-by-Row Approximation

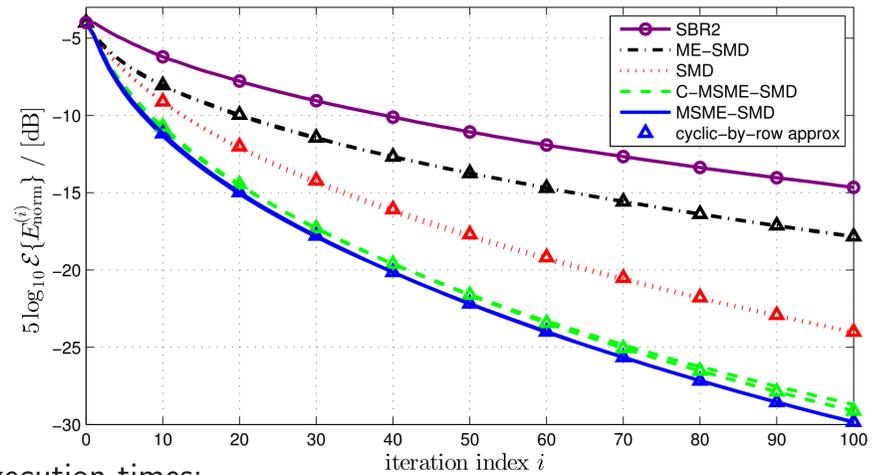
Idea: Using a limited number of sparse Jacobi transformations gives a good approximation to the full EVD but at a reduced cost.

$$\prod_{n=1}^N Q^{(i,n)} = \prod_{n=1}^N \begin{bmatrix} \mathbf{I}_{1,n} & & & \\ & \cos \varphi^{(i,n)} & \dots & e^{j\vartheta^{(i,n)}} \sin \varphi^{(i,n)} \\ & \vdots & \mathbf{I}_{2,n} & \vdots \\ & -e^{-j\vartheta^{(i,n)}} \sin \varphi^{(i,n)} & \dots & \cos \varphi^{(i,n)} \\ & & & & \mathbf{I}_{3,n} \end{bmatrix}$$

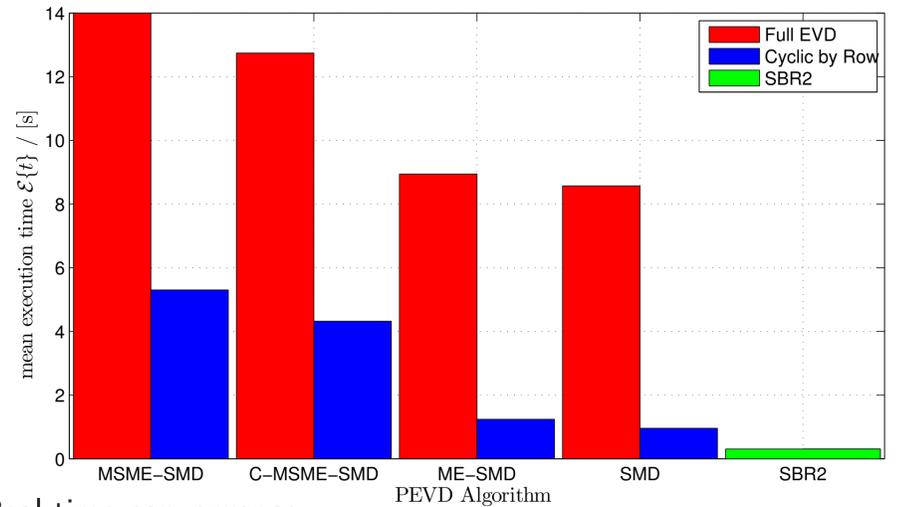
- ▶ The Cyclic-by-Row approximation does an element-wise “sweep” of the upper triangular region of the zero lag slice.
- ▶ Through experimentation a single cyclic-by-row sweep proved to have the best cost/performance trade-off.

Results

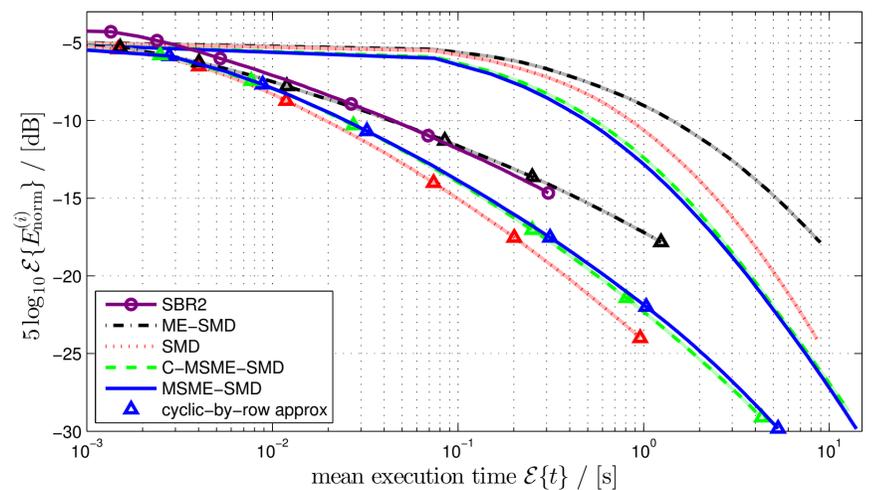
Algorithm convergence:



Execution times:



Real time convergence:



Conclusions

By replacing the costly EVD step with the cyclic-by-row approximation the execution times of the SMD algorithms have reduced by over 60% with minimum affect on overall algorithm convergence.

When the cyclic-by-row approximation is used the majority of the SMD algorithms have a better real time convergence than SBR2, which is a significant improvement compared to when the full EVD is used.

References

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- [2] J. G. McWhirter et al. An EVD Algorithm for Para-Hermitian Polynomial Matrices. *IEEE Transactions on Signal Processing*, 55(5):2158–2169, May 2007.
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- [4] S. Redif et al. An approximate polynomial matrix eigenvalue decomposition algorithm for para-hermitian matrices. In *11th IEEE Int. Symp. Sig. Proc. Inf. Tech.*, pp. 421–425, Dec. 2011.