Game theoretic power allocation for a multistatic radar network in the presence of estimation error

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SSPD Conference, Edinburgh

2014, 8-9 September

Outline

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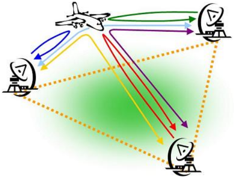
1. Introduction
2. Game Theoretic Power Allocation Scheme
3. SDR Estimation
4. Simulation Results
5. Conclusion

Introduction



- We consider a scenario of a radar network that consists of clusters of MIMO radars.
- Motivated by the work of Bacci et al. we examined the possibility of optimising the transmission power for each radar while attaining a specific detection performance.
- The aim is to enable the clusters in the network to operate independently, without the necessity of centralised control or any coordination among them.
- Using non cooperative game theory we have developed an algorithm for distributed power allocation for this scheme.
- Through simulations we have shown the convergence of the game theoretic algorithm to an equilibrium for different network topologies.

G. Bacci and L. Sanguinetti and M.S. Greco and M. Luise, "A game-theoretic approach for energy-efficient detection in radar sensor networks", *2012 IEEE 7^{th} Sensor Array and Multichannel Signal Processing Workshop (SAM)*, pp.157–160, 2012

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Multiple-Input Multiple-Output (MIMO) Radars

MIMO Radar

Multiple-Input Multiple-Output refers to a radar architecture that uses multiple antennas for both transmitting and receiving tasks. The antennas can be collocated or widely separated.

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MIMO radar

- allows the transmission of independent signals which lead to valuable information on the spatial diversity of the target
- offers flexible time-energy management (detection of multiple targets in one scan)
- supports the detection of slow-moving targets by using the Doppler information from different directions.

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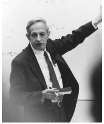
The solution of a game is a systematic description of the outcomes that may arise in a family of games.



Nash Equilibrium is the action profile such that no player can profitably deviate from their strategy.

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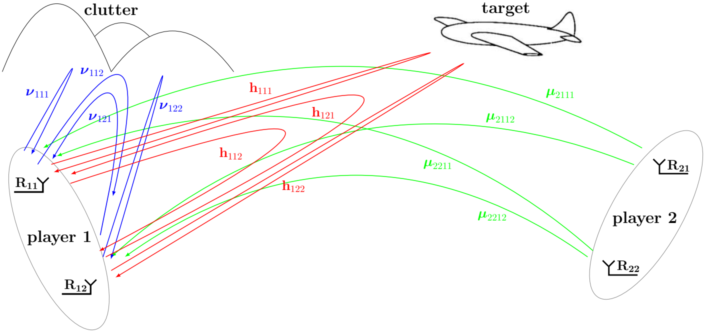
Model description

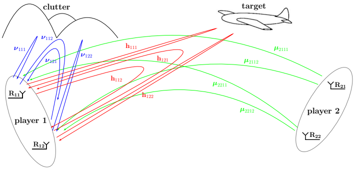
- A radar network consisting of a set $C = \{C_1, \ldots, C_K\}$ of clusters where each cluster k is formed by $C_k = \{R_{k1}, \ldots, R_{kM}\}$ MIMO radars.
- The task of each cluster is target detection under power constraints: use minimum transmission power while attaining a specific signal-to-disturbance ratio (SDR).
- Cooperation is assumed only among the radars within each cluster.
- The clusters should avoid causing deliberate interference to the other clusters.



Game theoretic formulation

Signal-to-disturbance ratio:

$$\mathsf{SDR}_{ki} = \frac{\sum_{j=1}^{M} h_{kji} p_{kj}}{\sum_{j=1}^{M} \nu_{jki} p_{kj} + \sum_{\substack{\ell=1 \\ \ell \neq k}}^{K} \sum_{j=1}^{M} \mu_{\ell jki} p_{\ell j} + \sigma_n^2}$$



Payoff Function

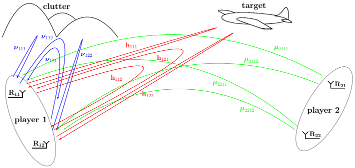
The payoff function of player k is defined as

$$u_k(\mathbf{p}_{-k}, \mathbf{p}_k) = \sum_{i=1}^{M} p_{ki}.$$

Game theoretic formulation (continued)

The game is defined as the tuple

$$\mathcal{G} =< C, \{\mathcal{P}_k\}_{k\in\{1,\dots,K\}}, \{u_k\}_{k\in\{1,\dots,K\}} >$$



The best response of the k^{th} player to the current strategy of all players is given by the following constrained optimisation:

$$\min_{\mathbf{p}_k\in S_k(\mathbf{p}_{-k})} u_k(\mathbf{p}_{-k},\mathbf{p}_k) = \min_{\mathbf{p}_k\in S_k(\mathbf{p}_{-k})} \sum_{i=1}^{M} p_{ki}$$

$$\text{s.t.} \quad \mathsf{SDR}_{ki} = \frac{\sum\limits_{j=1}^{M} h_{kji}p_{kj}}{\sum\limits_{j=1}^{M} \nu_{jki}p_{kj} + \sum\limits_{\substack{\ell=1\\ \ell\neq k}}^{K}\sum\limits_{j=1}^{M} \mu_{\ell jki}p_{\ell j} + \sigma_n^2} \geq \gamma_t \quad \forall i = 1,\dots,M$$

Outline

SDR Estimation

The return signal received by radar R_{ki} is given by

$$\mathbf{x}_{ki} = \sum_{j=1}^{M} \alpha_{kji}\widetilde{\mathbf{s}}_{kj} + \mathbf{i}_{ki} + \mathbf{d}_{ki}$$

- $\alpha_{kji} \sim \mathcal{CN}(0, h_{kji}p_{kj})$ (channel gain in the direction of target)
- h_{kji} (average signal propagation loss)
- p_{kj} (transmission power)
- $\mathbf{i}_{ki}$ (interference)
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where

$$\frac{\sum\limits_{j=1}^{M} |\widetilde{\mathbf{s}}_{kj}^H \mathbf{x}_{ki}|^2}{N} \approx \sum_{j=1}^{M} |\alpha_{kji}|^2 N \quad \text{and} \quad \frac{||\mathbf{x}_{ki}||^2}{N} \approx \sum_{j=1}^{M} |\alpha_{kji}|^2$$

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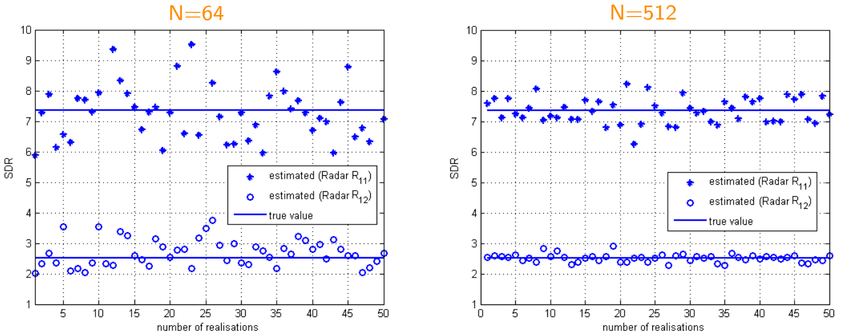
SDR Estimation (cont'd)

- At each time step the radars receive N signal return samples
- The network is formed by two clusters with two radars per cluster.
- The transmission power for all radars is fixed to 0.1W, and the noise power is set to 0.01W.

True vs Estimated SDR



Outline

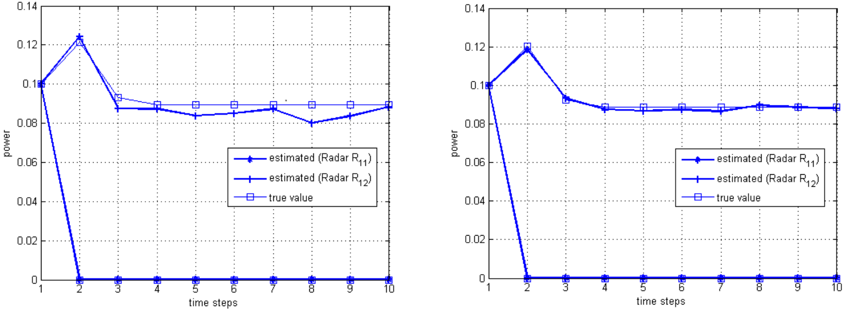
Simulation Results I

- Network of two clusters, with two radars per cluster

Power allocation for cluster 1

N=64

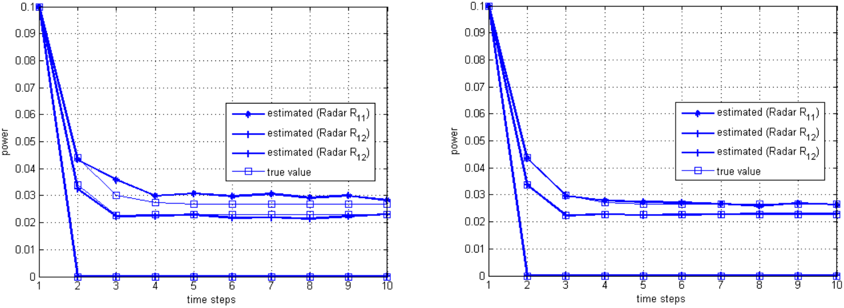
N=512



Simulation Results II

- Network of two clusters, with three radars per cluster

Power allocation for cluster 1



N=64

N=512

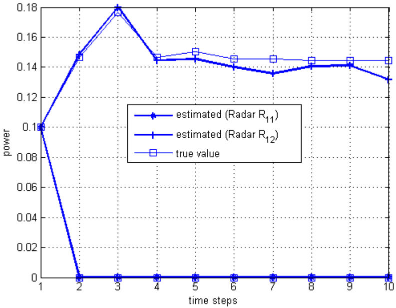
Simulation Results III

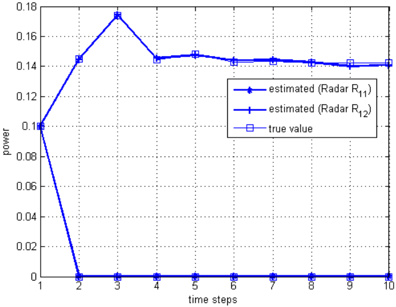
- Network of three clusters, with two radars per cluster

Power allocation for cluster 1

N=64

N=512





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Summary

- We proposed a game theoretic power allocation scheme for a MIMO radar network where the radars are grouped into clusters and operate in a friendly but noncooperative environment.
- We modelled our scheme using the generalised Nash game with the players employing the best response approach.
- We demonstrated the convergence to the Nash equilibrium through simulation, assuming a network of different topologies.
- We investigated the performance of the game theoretic algorithm in the presence of estimation error and showed that the proposed algorithm has the potential to converge to an equilibrium and achieve the target SDR, even in the presence of an estimation error.

Thank you!