Sparse Methods in Radar Signal Processing

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Presented at: the 2014 Sensor Signal Processing for Defence Conference, Edinburgh, Scotland

Sparse Inferences about Scotland

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- It never rains in Edinburgh

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- The letter 's' is subject to P_M and P_{FA}
  - P_M: Defence vs Defense
  - P_{FA}: Optimization vs Optimisation

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Sparse Inferences about Scotland

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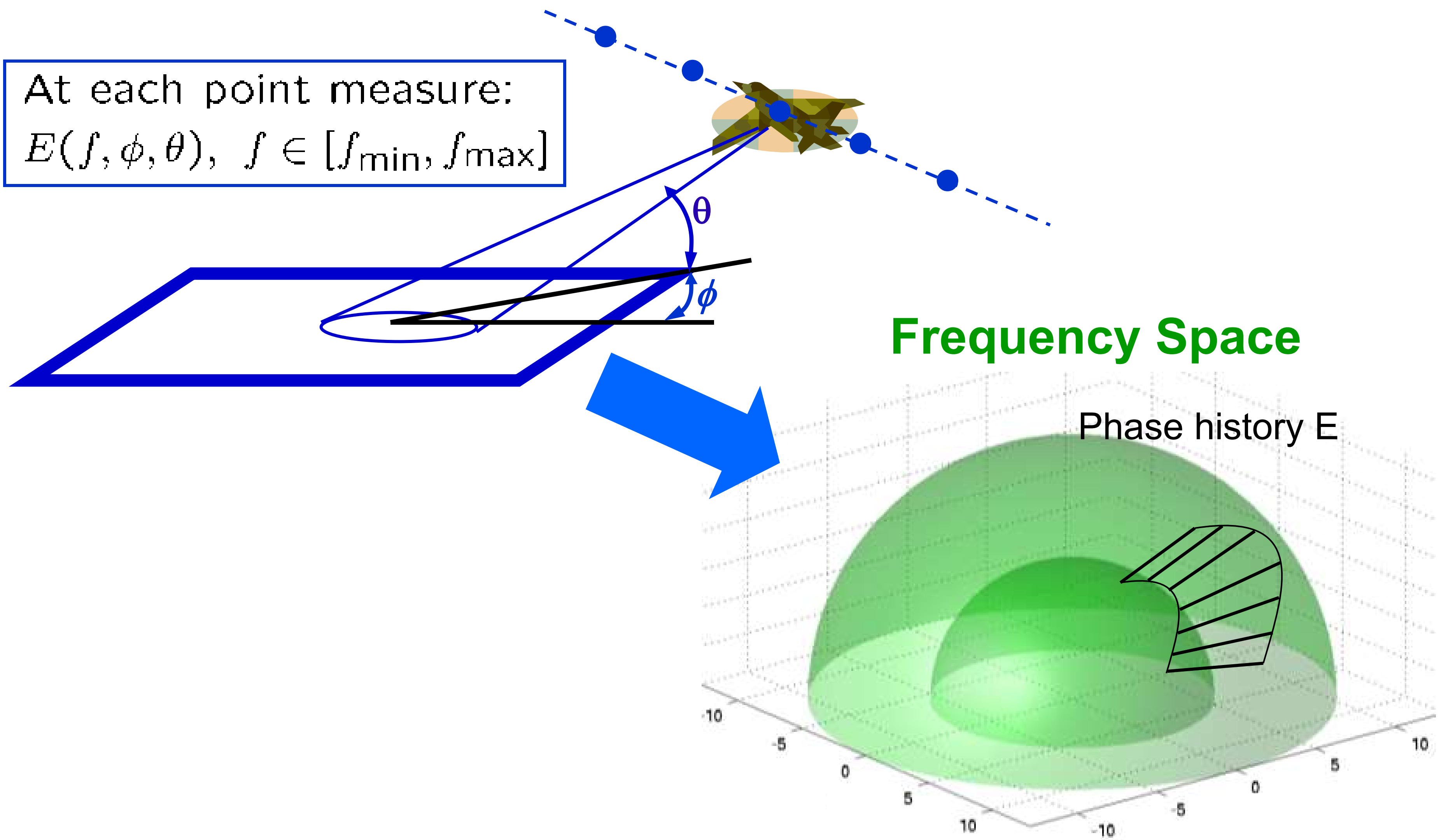
  - ~~P_M~~: P_{FA}! Defence vs Defense
  - ~~P_{FA}~~: P_M! Optimization vs Optimisation

- If a roundabout doesn't have trees or grass on it, it is perfectly okay to drive right over it.
  - My apologies to any of you who were approaching a roundabout while I was driving through!

Context

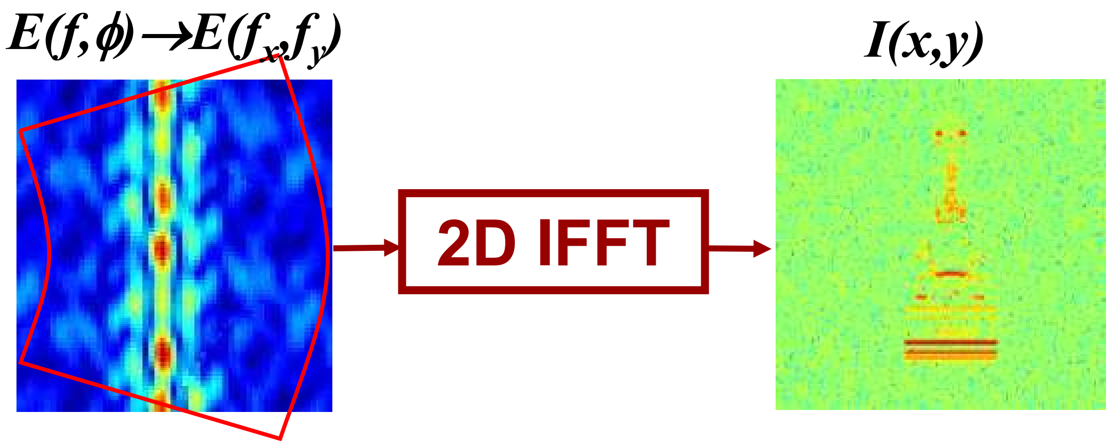
- Advances in digital processing are enabling revolutionary opportunities for radar signal processing
  – Sophisticated radar image/volume reconstructions
  – Multi-function radars that can simultaneously perform imaging, detection, moving object tracking and recognition, etc.
  – Persistent sensing over space and time
  – Combined sensing and communication
  – Estimation/inference with uncertainty analysis
- Challenges
  – Very large data, processing, and communications tasks
  – Traditional models for radar backscattering may not apply over wide angles

SAR Data Collection



SAR Image Formation

- Traditional approach: tomography



- Tomographic image $I(x,y)$ is a **matched filter** for an isotropic point scatterer at location (x,y). [Rossi+Willsky]

Linear Algebra Formulation

- Measurements y (M £ 1) : phase history data as a function of (f,az,el)
- Reconstruction: x (N £ 1): set of (x,y) or (x,y,z) locations with significant radar scattering energy

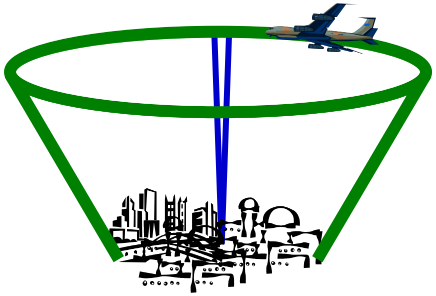
$$y = Ax + \nu$$

$$A = \left[e^{-j(k_{x,m}x_n + k_{y,m}y_n + k_{z,m}z_n)} \right], \quad M \times N$$

Matched filter:

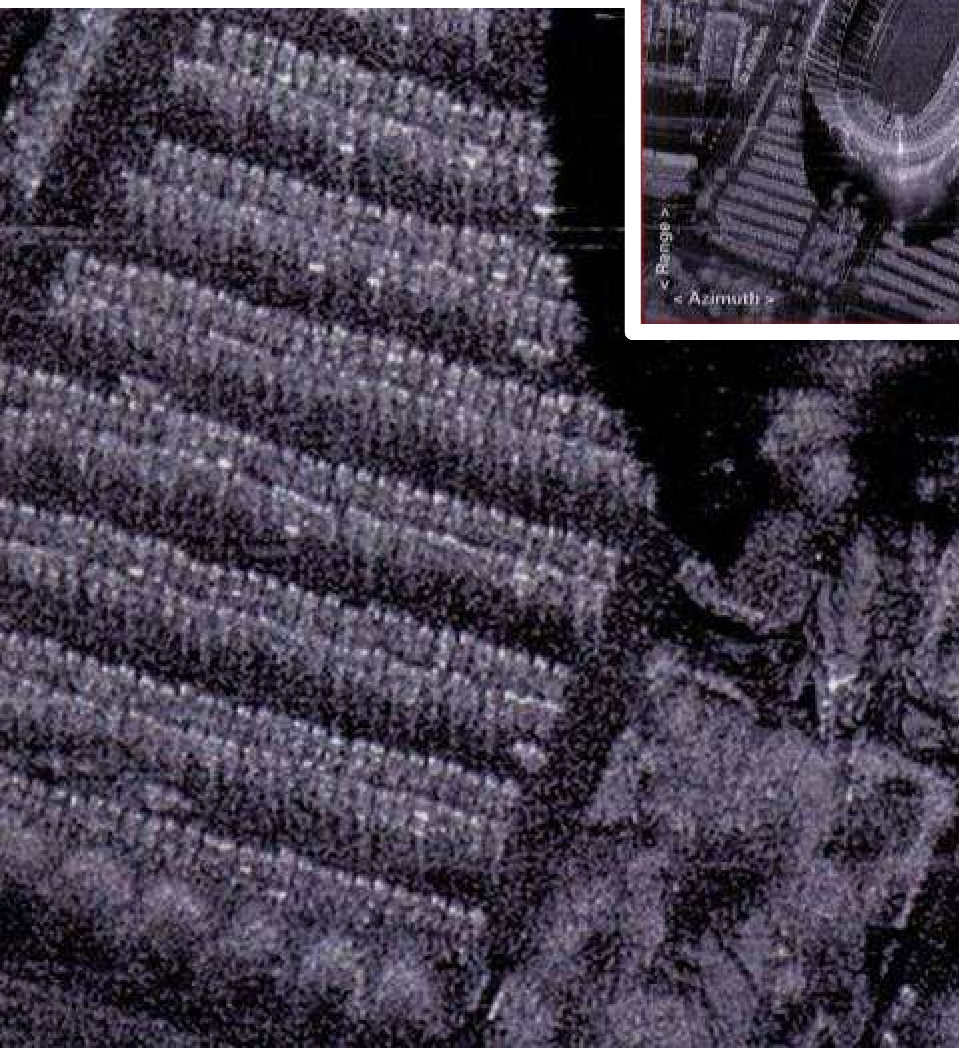
$$\hat{x} = A^H y$$

Example: Ohio Stadium

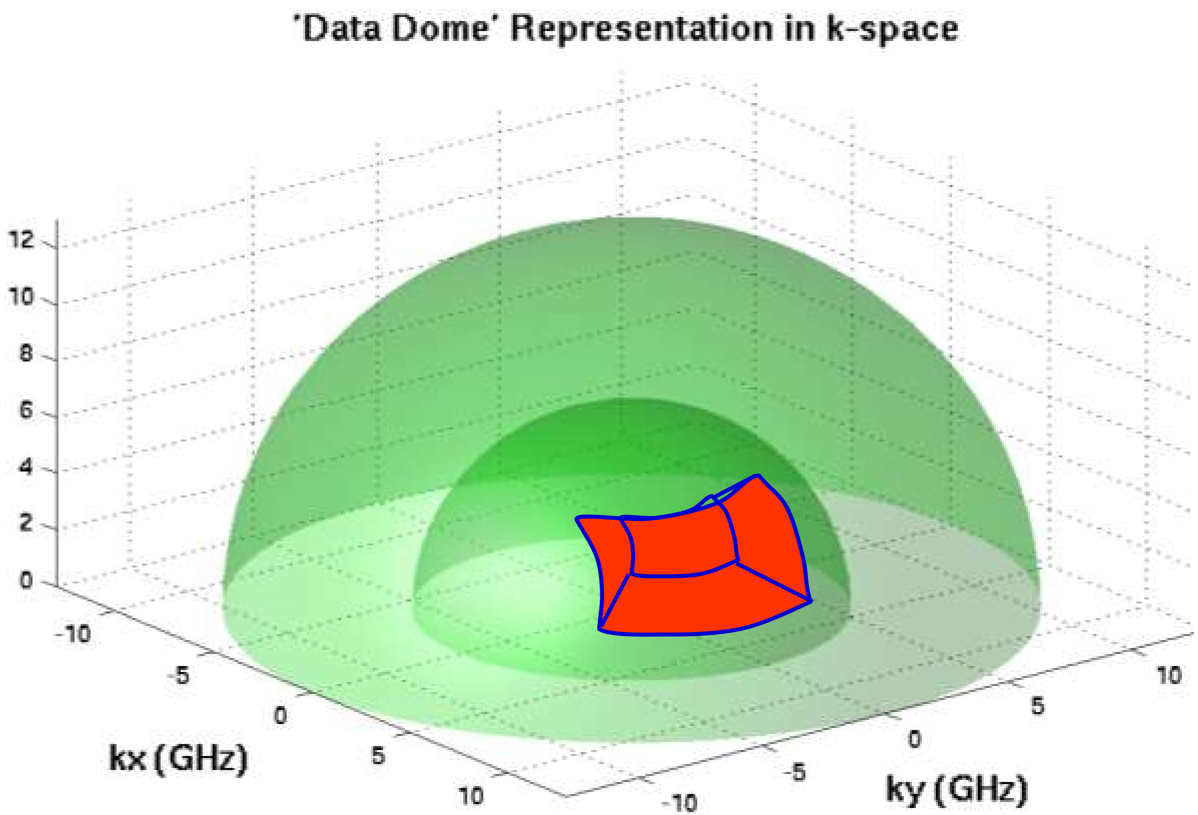


X-Band Radar
3° aperture
1ft x 1ft res

SAR Image Detail



3D Reconstruction



- Massive data size and processing needs
- Filled aperture is difficult to collect

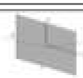
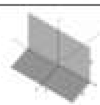
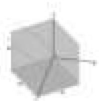
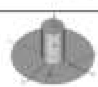
Can Sparsity Play a Role?

- At high frequencies, radar backscatter is well-modeled as a sum of responses from canonical scattering terms.
- EM scattering theory provides a rich characterization of backscatter behavior as a function of object shape
  - Azimuth, elevation, frequency dependence
  - Polarization dependence
  - Phase response - range
- This scattering theory suggests that the radar response may be sparse in some representations
  - Sparse reconstruction
  - Parametric modeling

Scattering Model

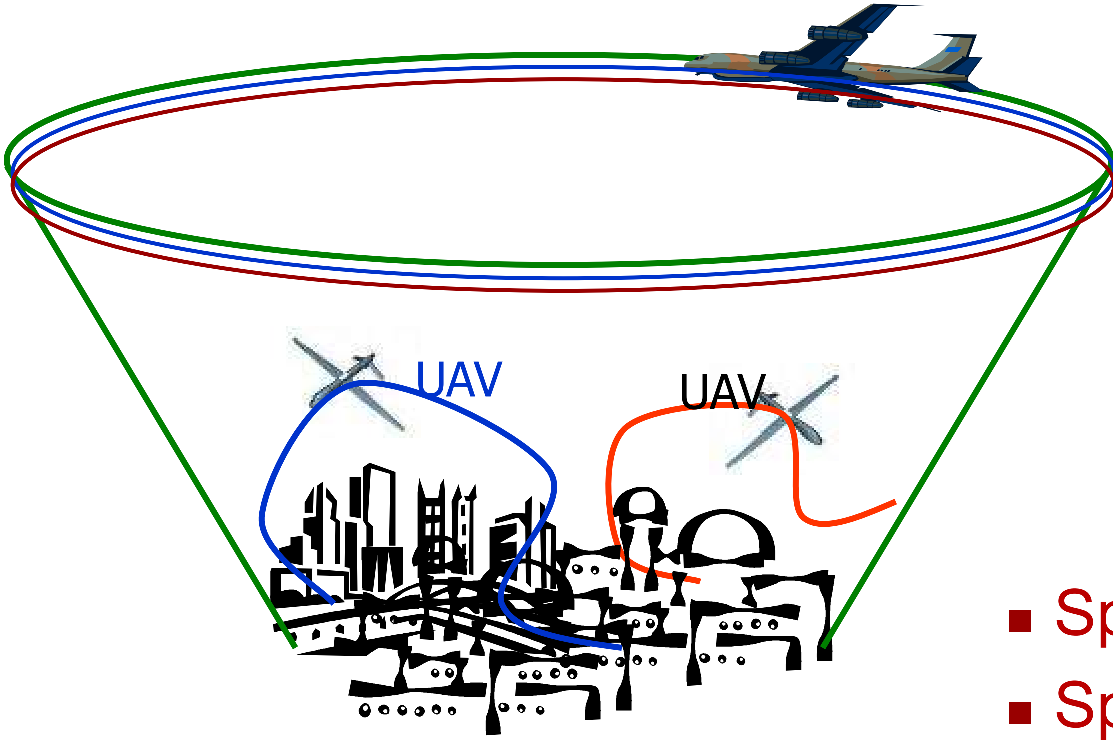
$$S(f,\phi,\theta) = \begin{bmatrix} A_{HH} & A_{HV} \\ A_{VH} & A_{VV} \end{bmatrix} \left(\frac{jf}{f_c}\right)^{\gamma} M(\phi,\theta)\, e^{j\frac{4\pi f}{fc}\Delta R(x,y,z;a)}$$

Polarization Dependence — **Frequency Dependence** — **Aspect Dependence** — **Location Dependence**

Canonical Shape Type Γ	Icon	Polarization Type β	Amplitude Response $M_\Gamma(k,\hat{\phi},\hat{\theta};\Theta)$	Calibration Factor A	Range Offset ΔR_r
Plate		odd	$M_{\text{plate}}(\Theta_{\text{plate}}) = \frac{jkA}{\sqrt{\pi}}\,\text{sinc}\left[kL\sin\hat{\phi}\cos\hat{\theta}\right]\,\text{sinc}\left[kH\sin\hat{\theta}\right]$, $\hat{\phi}\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	LH	0
Dihedral		even	$M_{\text{dih}}(\Theta_{\text{dih}}) = \frac{jk}{\sqrt{\pi}}A\,\text{sinc}\left[kL\sin\hat{\phi}\cos\hat{\theta}\right] \times \begin{cases}\sin\hat{\theta}, & \hat{\theta}\in[0,\frac{\pi}{4}] \\ \cos\hat{\theta}, & \hat{\theta}\in[\frac{\pi}{4},\frac{\pi}{2}]\end{cases}, \quad \hat{\phi}\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$2LH$	0
Trihedral		odd	$M_{\text{tri}}(\Theta_{\text{tri}}) = \frac{jkA}{2\sqrt{\pi}} \times \begin{cases}-\cos\left(\hat{\phi}-\frac{\pi}{4}\right), & \hat{\phi}\in[\frac{-\pi}{4},0] \\ \sin\left(\hat{\phi}-\frac{\pi}{4}\right), & \hat{\phi}\in[0,\frac{\pi}{4}]\end{cases} \times \begin{cases}\sin\left(\hat{\theta}+\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)\right), & \hat{\theta}\in\left[0,\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)\right] \\ \cos\left(\hat{\theta}+\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)\right), & \hat{\theta}\in\left[\tan^{-1}\left(\frac{1}{\sqrt{2}}\right),\frac{\pi}{2}\right]\end{cases}$	$2\sqrt{3}H^2$	0
Cylinder		odd	$M_{\text{cyl}}(\Theta_{\text{cyl}}) = jk\sqrt{\cos\hat{\phi}}A\,\text{sinc}\left[kL\sin\hat{\phi}\right]$, $\hat{\phi}\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$L\sqrt{r}$	$2r\cos\phi$
Top-hat		even	$M_{\text{top}}(\Theta_{\text{top}}) = A\sqrt{jk} \times \begin{cases}\sin\hat{\theta}, & \hat{\theta}\in[0,\frac{\pi}{4}] \\ \cos\hat{\theta}, & \hat{\theta}\in[\frac{\pi}{4},\frac{\pi}{2}]\end{cases}$	$\sqrt{\frac{8r}{\sqrt{2}}}H$	$2r\cos\theta$
Sphere		odd	$M_{\text{sphere}}(\Theta_{\text{sphere}}) = A\sqrt{\pi}r$	1	$2r$

Jackson & RLM: *2009*

Persistent, Wide-Angle Radar



- Sparsity in sensing
- Sparsity in reconstruction
  - Compressive sensing
  - Other sparse reconstruction techniques
  - Parametric modeling

Sparse Reconstruction

Sparsity

- Measurements y (M £ 1) : sparse sampling of full (f,az,el) radar measurement space
- Reconstruction: x (N £ 1): sparse set of (x,y,z) locations with significant radar scattering energy

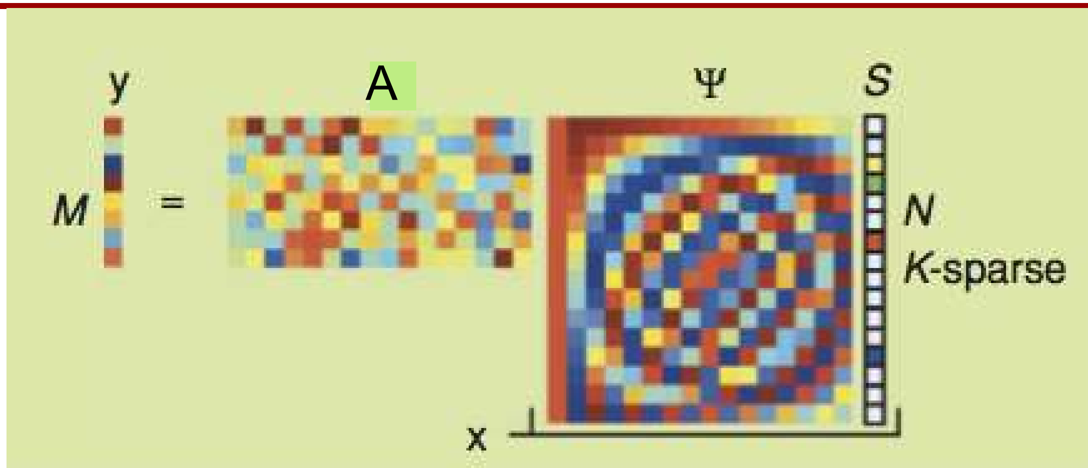
$$y = Ax + \nu$$

$$A = \left[e^{-j(k_{x,m}x_n + k_{y,m}y_n + k_{z,m}z_n)} \right], \quad M \times N$$

Sparse reconstruction:

$$\hat{x} = \arg\min_x \; ||y - Ax||_2^2 + \lambda ||x||_p^p \quad p \leq 1$$

Compressive Sensing



$$y = Ax + \nu$$

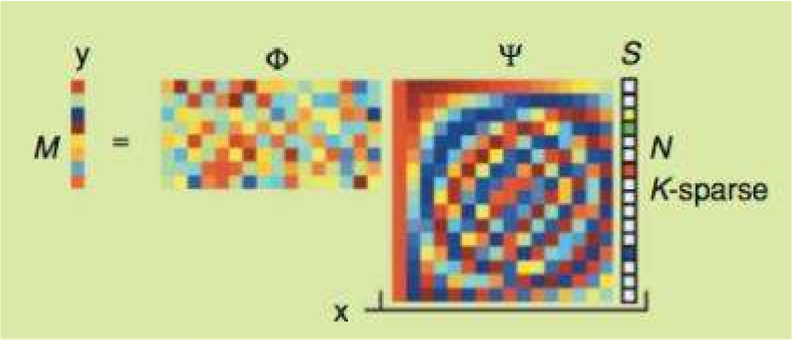
A satisfies the Restricted Isometry Property (RIP)

$$\hat{x} = \|y - Ax\|_2^2 + \lambda \|x\|_1$$

Compressive Sensing

- **Discrete linear model**
- **ℓ_1 regularization (=convex problem)**
- **Provable performance guarantees**

Compressive Sensing: Hype or Help?



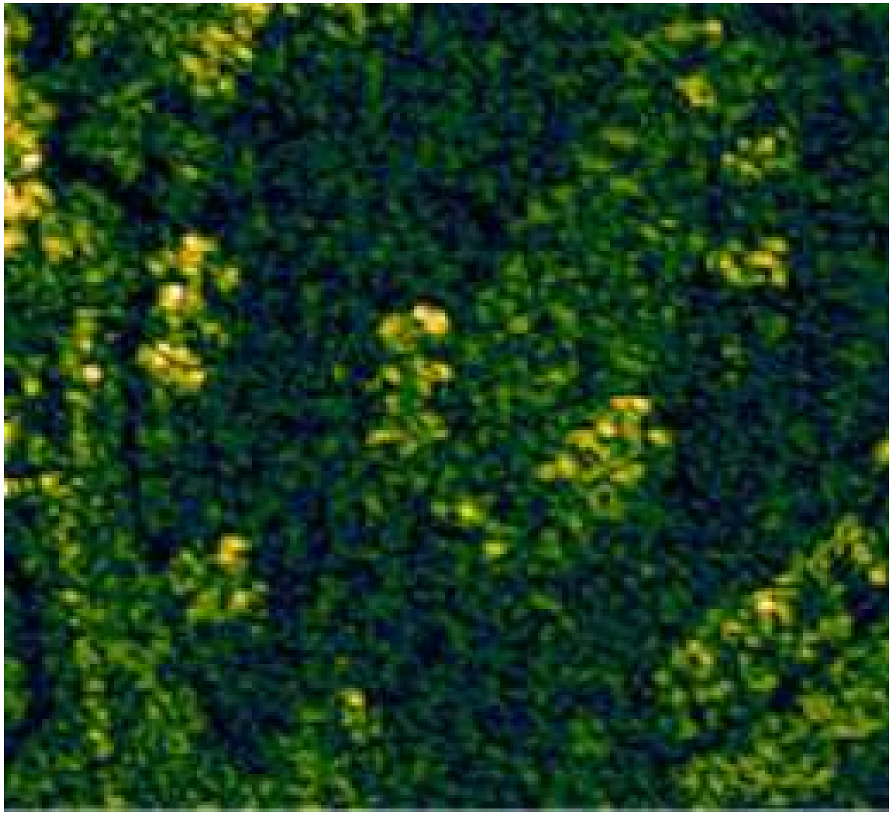
Does compressive sensing apply to radar?

- Hype or help?
- Bandwagon or breakthrough?
- Satan or salvation?
- Fraud or foundation?

From: L. Potter, Optical Society of America Incubator, April 2014

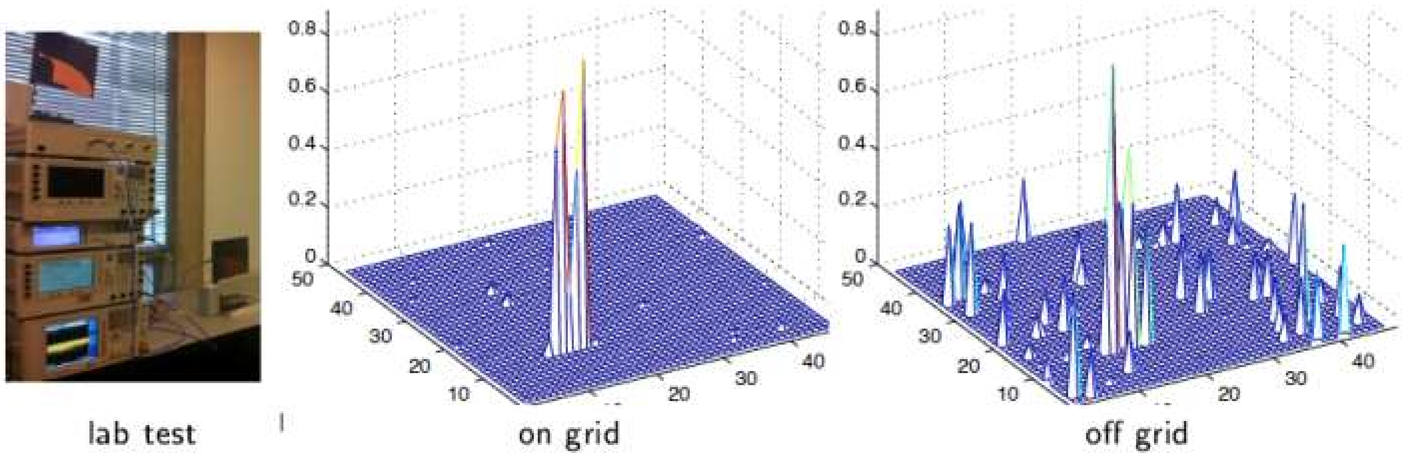
Bah! Humbug! Part 1

- Radar signals aren't compressible in many applications.
- For air-to-ground surveillance, sensor data has *high entropy*



Bah! Humbug! Part 2

- CS orthodoxy assumes ranges, angles, velocities are *discretized* to a sample grid – yet these parameters are *continuous-valued*.
- Basis mismatch leads to loss of sparsity; oversampled grids destroy low coherence

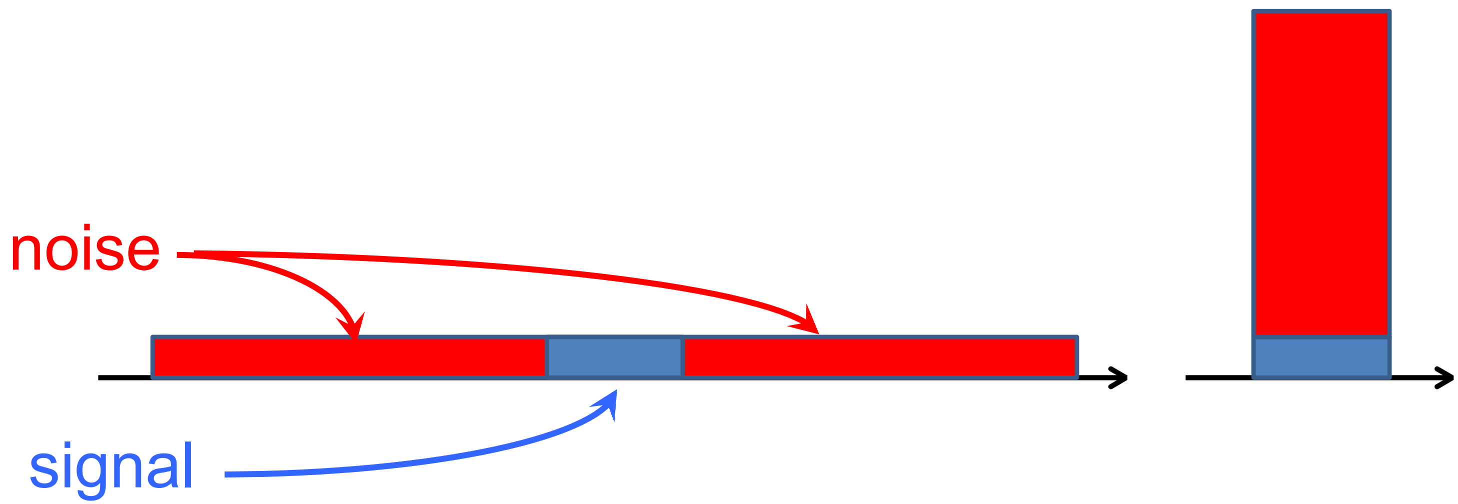


Benchtop X-band result using ℓ_1 with chirp waveforms and 47:1 compression.

Presented at: the 2014 Sensor Signal Processing for Defence Conference, Edinburgh, Scotland

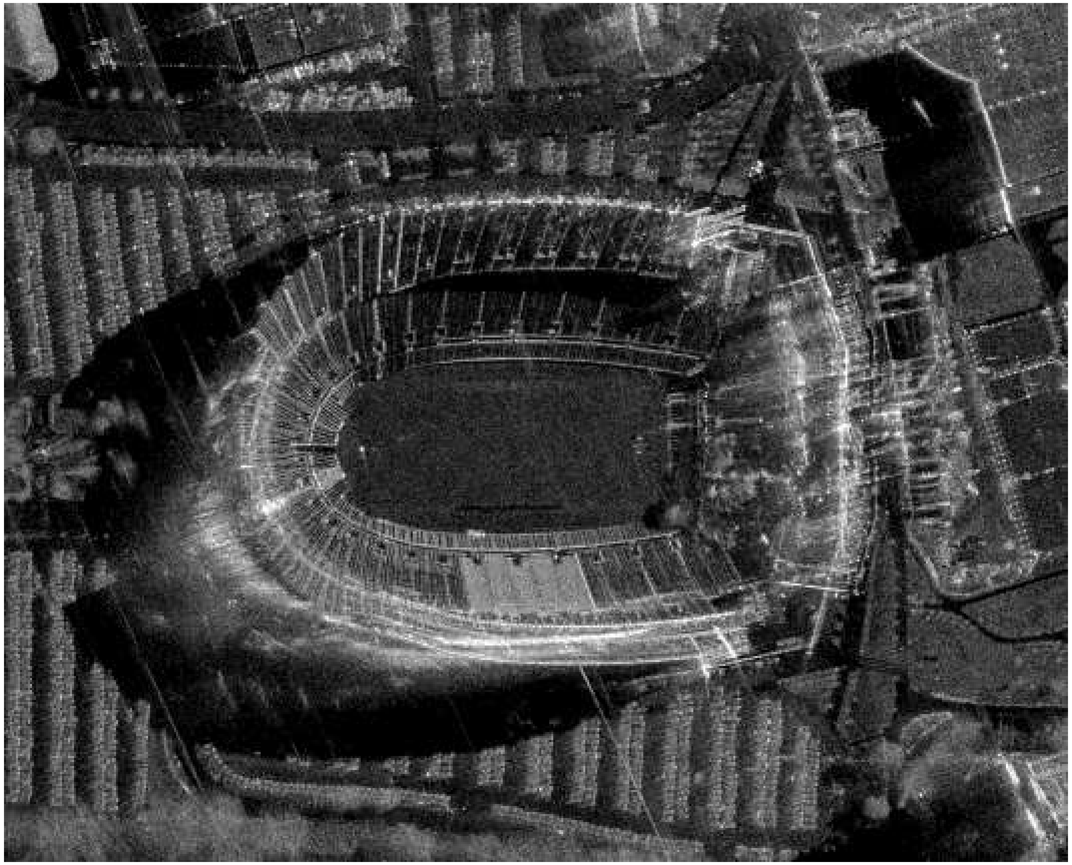
Bah! Humbug! Part 3

- RF receiver noise power, cost, and power consumption scale with the precision of sample timing, not the average #samples per unit time.



Bah! Humbug! Part 4

- Linear Processing: Image analysts understand and accept the *structured and predictable* artifacts of linear processing
- Nonlinear processing artifacts are unpredictable and foreign



Bah! Humbug! Part 5

- CS is an imposter: it's been around for seven decades or more…

THE BELL SYSTEM

TECHNICAL JOURNAL

VOLUME XXXIX — JULY 1960 — NUMBER 4

Copyright 1960, American Telephone and Telegraph Company

The Theory and Design of Chirp Radars

By J. R. KLAUDER, A. C. PRICE,
S. DARLINGTON and W. J. ALBERSHEIM

(Manuscript received April 5, 1960)

United States Patent [19]

Stromswold et al.

[54] **COMPRESSIVE RECEIVER**

[75] Inventors: **Chester E. Stromswold**, Nashua; **John T. Apostolos**, Manchester, both of N.H.; **Robert P. Boland**, Malden; **Walter J. Albersheim**, Wayban, both of Mass.

[73] Assignee: **Sanders Associates, Inc.**, Nashua, N.H.

[21] Appl. No.: **871,297**

[22] Filed: **Jan. 23, 1978**

CS as Hype

Compressive Sensing is hype, suited to carnival criers, research funding chasers, and academic navel gazing.

As far as RF sensing is concerned, it belongs in a *dust bin.*

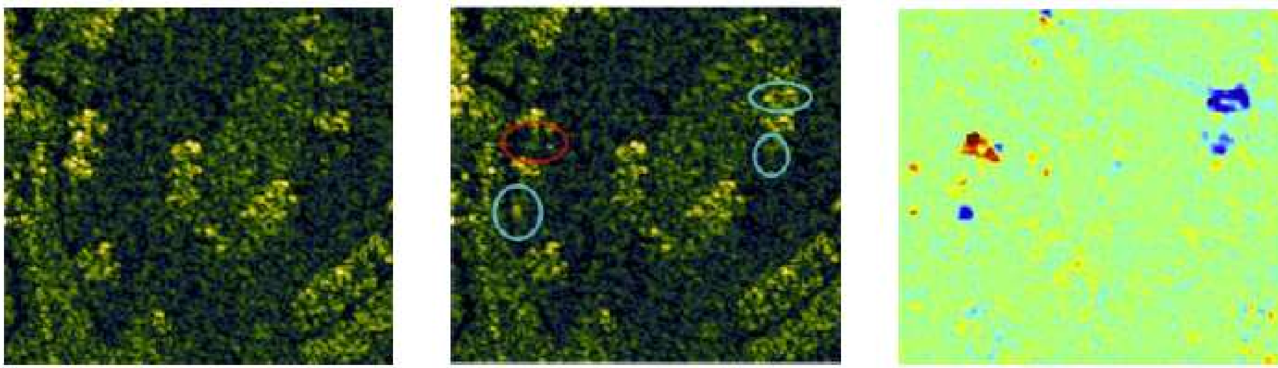
Rebuttal:

Why Compressed Sensing matters for practical radar

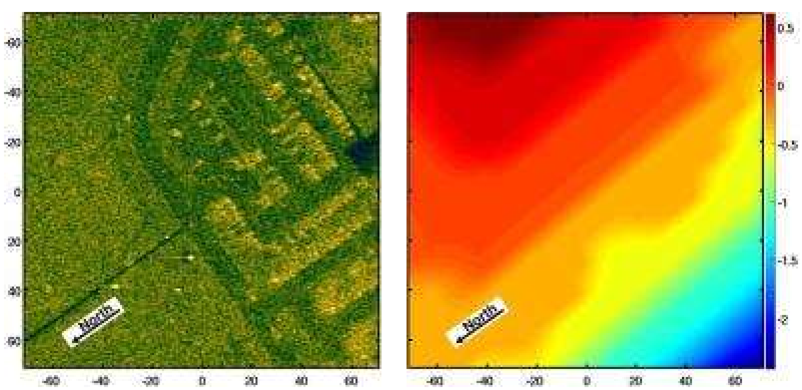
Rebuttal 1: Low Entropy

While radar images are on the whole high entropy, many applications have low entropy signals or components.

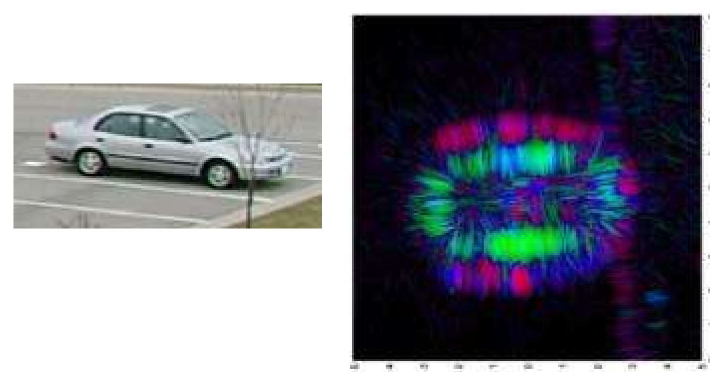
Change Detection



Autofocus (phase tracking)



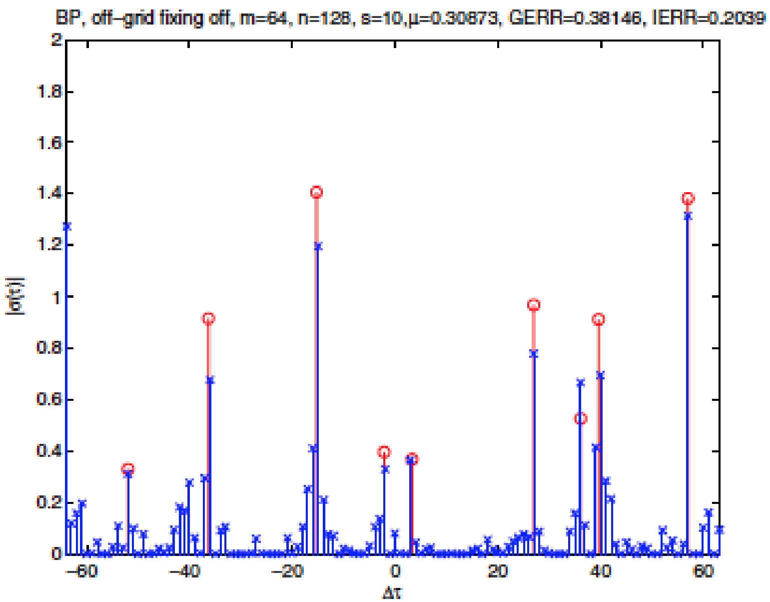
Target Chips



Rebuttal 2:

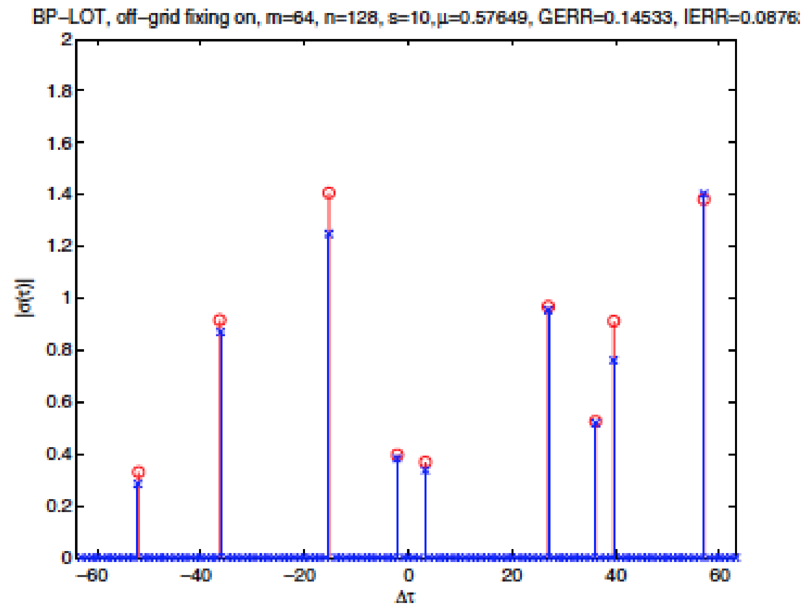
- Recent advances effectively address grid quantization
  - Fannjiang
  - Austin

BP



vs

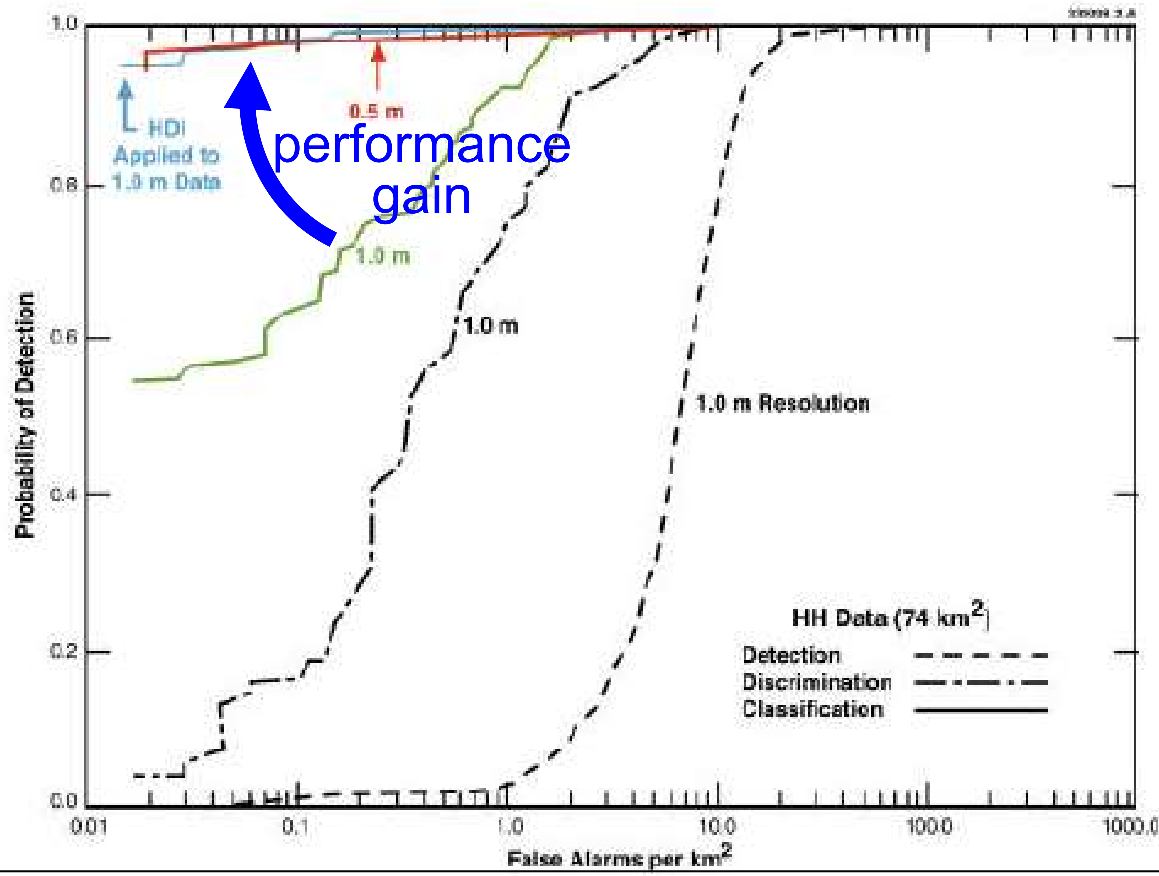
BP-LOT



From: A. Fannjiang and H-C Tseng, “Compressive radar with off=grid targets: a perturbation approach,” *Inverse Problems* **29** (2013) 054008.

Rebuttal 3: Performance Gains

- In *quantitative* ATR performance, ~2x effective resolution enhancement is observed using sparse recovery methods.



Ka Band 1-meter SAR ROC curves for the 74 km^2 Stockbridge NY and Ayer MA clutter data and 192 TEL target images (Lincoln Lab, 1996)

Rebuttal 4: New insights, algorithms

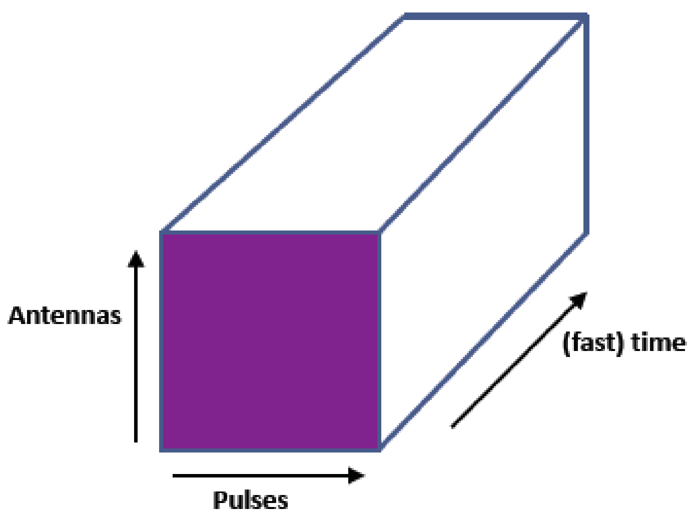
- Semi-definite programming formulation gives tractable computation
  - Impressive gains in speed, convergence in a few short years
- Convex formulation yields provable finite-sample performance guarantees
- Seamlessly tackles model order selection

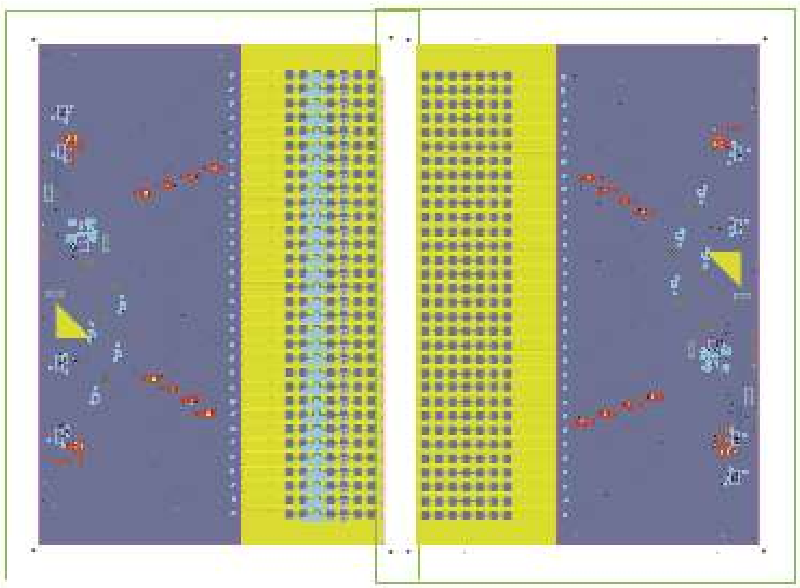
Algorithm	Error	Time (sec)
Singular value thresholding [Cai/Candes/Shen08]	3.4e-4	877
Dual method [Ganesh/Wright/Wu/Chen/Ma'09]	1.6e-5	177
Accelerated proximal gradient [Ganesh/Wright/Wu/Chen/Ma'09]	1.8e-5	8
Alternating direction methods [Yuan/Yang'09]	2.2.e-5	5
Inexact augmented Lagrange method [Lin/Chen/Wu/Ma'09]	4.3e-8	2
Bilinear generalized AMP [Schniter/Parker/Cevher'12]	3.8e-8	1

Recovery of 400x400 rank-20 matrix corrupted by 5%-sparse amplitudes uniform on [-50,50]

Rebuttal 5: multi-mode enabler

- Sampling across space (antenna arrays) and slow-time (pulses) provide avenues for compression beyond stretch processing
- Compression across antennas and pulses provides flexibility for multi-mode RF system operation



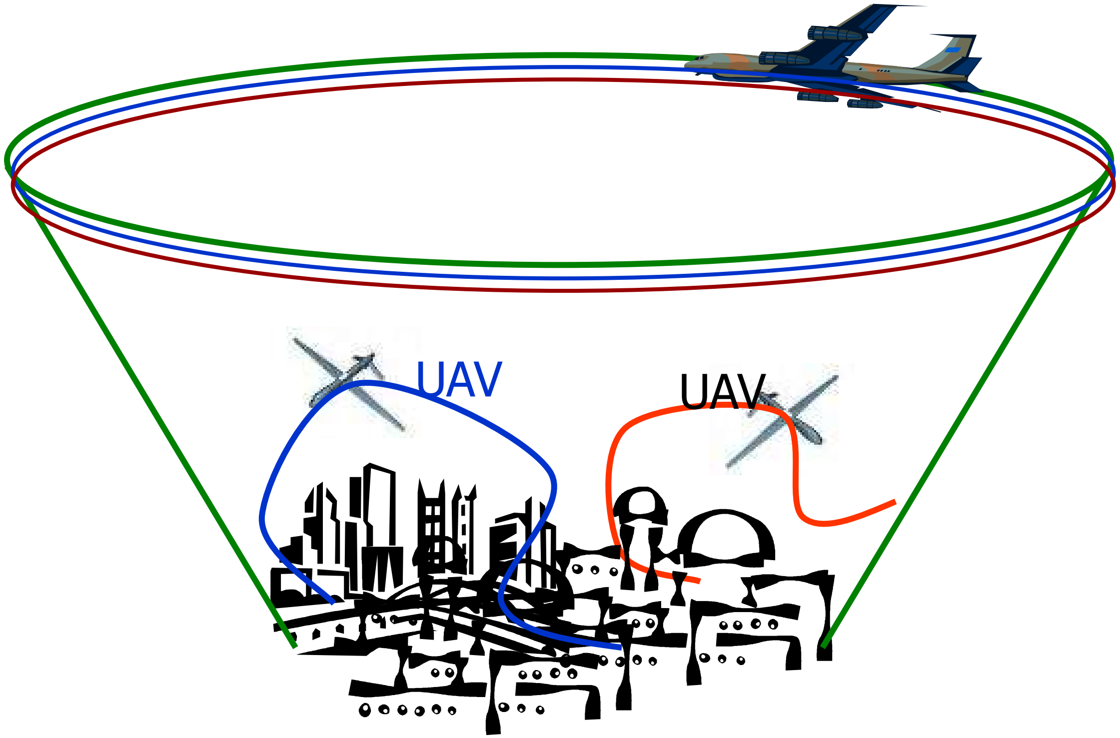


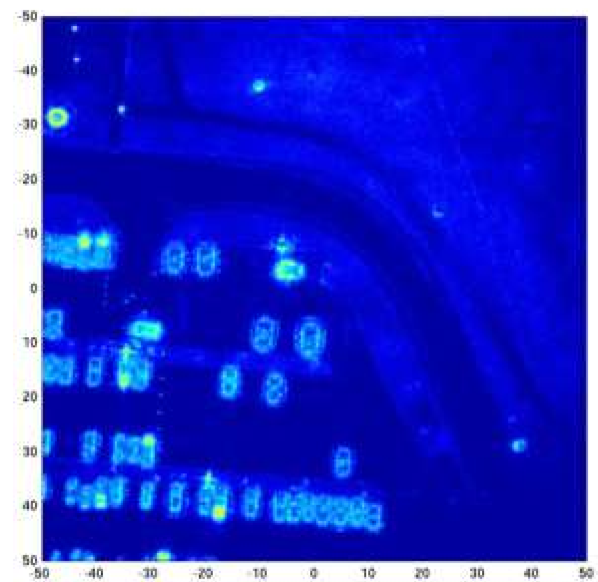
The Front Porch

The *accessibility* and *popularity* of compressive sensing provides a format for rich *cross-disciplinary interactions* and an invitation for practitioners to *reconsider data acquisition and nonlinear processing*.

- Vocabulary of *linear algebra* to consider inverse problems and estimation tasks
- Invitation to consider *signal structure* or parsimony beyond bandlimitedness
- Invitation to consider *non-uniform sampling* strategies
- Good *convex programming* codes.

How Can Sparsity Play a Role?





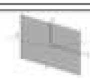
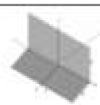
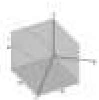
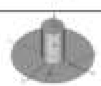
Persistent Sensing enables:

- ***High resolution, volumetric imaging of stationary objects and scenes***
- ***Continuous tracking of moving objects***

Scattering Model

$$S(f,\phi,\theta) = \begin{bmatrix} A_{HH} & A_{HV} \\ A_{VH} & A_{VV} \end{bmatrix} \left(\frac{jf}{f_c}\right)^{\gamma} M(\phi,\theta)\, e^{j\frac{4\pi f}{f_c}\Delta R(x,y,z;a)}$$

Polarization Dependence | **Frequency Dependence** | **Aspect Dependence** | **Location Dependence**

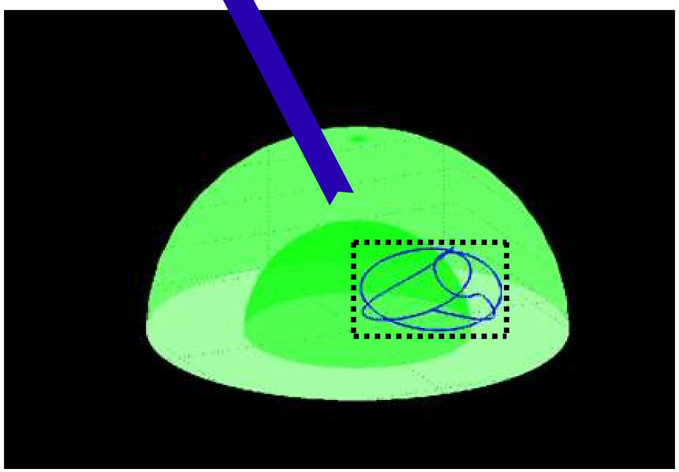
Canonical Shape Type Γ	Icon	Polarization Type β	Amplitude Response $M_\Gamma(k,\hat{\phi},\hat{\theta};\Theta)$	Calibration Factor A	Range Offset ΔR_r
Plate		odd	$M_{\text{plate}}(\Theta_{\text{plate}}) = \frac{jkA}{\sqrt{\pi}}\,\text{sinc}\left[kL\sin\hat{\phi}\cos\hat{\theta}\right]\text{sinc}\left[kH\sin\hat{\theta}\right]$ $\hat{\phi}\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	LH	0
Dihedral		even	$M_{\text{dih}}(\Theta_{\text{dih}}) = \frac{jk}{\sqrt{\pi}}A\,\text{sinc}\left[kL\sin\hat{\phi}\cos\hat{\theta}\right] \times \begin{cases}\sin\hat{\theta}, & \hat{\theta}\in\left[0,\frac{\pi}{4}\right] \\ \cos\hat{\theta}, & \hat{\theta}\in\left[\frac{\pi}{4},\frac{\pi}{2}\right]\end{cases},\quad \hat{\phi}\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$2LH$	0
Trihedral		odd	$M_{\text{tri}}(\Theta_{\text{tri}}) = \frac{jkA}{2\sqrt{\pi}} \times \begin{cases}-\cos\left(\hat{\phi}-\frac{\pi}{4}\right), & \hat{\phi}\in\left[\frac{-\pi}{4},0\right] \\ \sin\left(\hat{\phi}-\frac{\pi}{4}\right), & \hat{\phi}\in\left[0,\frac{\pi}{4}\right]\end{cases} \times \begin{cases}\sin\left(\hat{\theta}+\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)\right), & \hat{\theta}\in\left[0,\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)\right] \\ \cos\left(\theta+\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)\right), & \hat{\theta}\in\left[\tan^{-1}\left(\frac{1}{\sqrt{2}}\right),\frac{\pi}{2}\right]\end{cases}$	$2\sqrt{3}H^2$	0
Cylinder		odd	$M_{\text{cyl}}(\Theta_{\text{cyl}}) = jk\sqrt{\cos\hat{\phi}}A\,\text{sinc}\left[kL\sin\hat{\phi}\right] \quad \hat{\phi}\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$L\sqrt{r}$	$2r\cos\phi$
Top-hat		even	$M_{\text{top}}(\Theta_{\text{top}}) = A\sqrt{jk} \times \begin{cases}\sin\hat{\theta}, & \hat{\theta}\in\left[0,\frac{\pi}{4}\right] \\ \cos\hat{\theta}, & \hat{\theta}\in\left[\frac{\pi}{4},\frac{\pi}{2}\right]\end{cases}$	$\sqrt{\frac{8r}{\sqrt{2}}}H$	$2r\cos\theta$
Sphere		odd	$M_{\text{sphere}}(\Theta_{\text{sphere}}) = A\sqrt{\pi}r$	1	$2r$

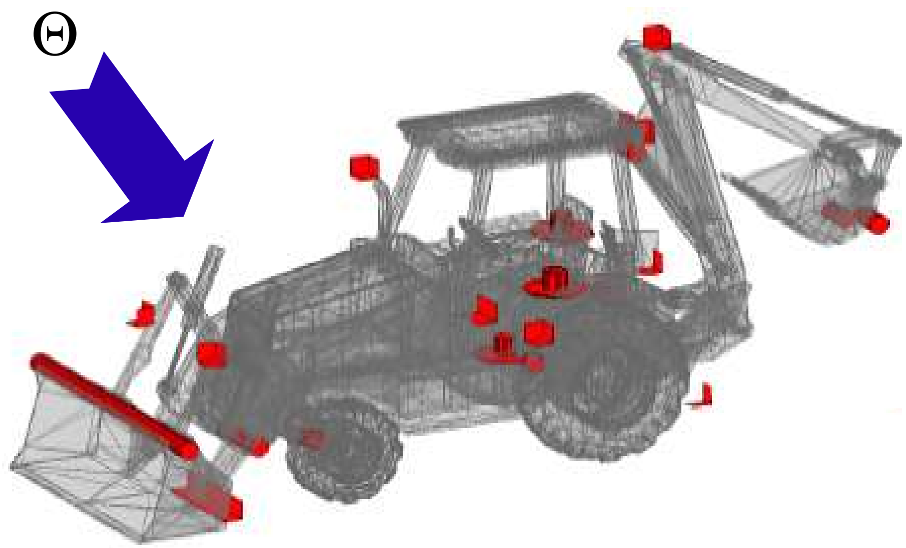
Jackson & RLM: *2009*

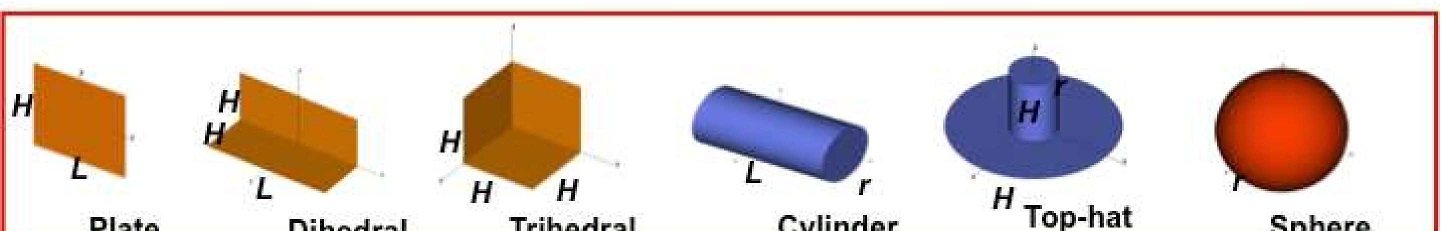
Parametric: Canonical Scattering Model

$$S(f,\phi,\theta) = \sum_{k=1}^{K} \begin{bmatrix} A_{HH} & A_{HV} \\ A_{VH} & A_{VV} \end{bmatrix}_k \left(\frac{jf}{f_c}\right)^{\gamma_k} M_k(\phi,\theta)\; e^{j\frac{4\pi f}{fc}\Delta R(x_k,y_k,z_k;a_k)}$$

Polarization Dependence | **Frequency Dependence** | **Aspect Dependence** | **Location Dependence**



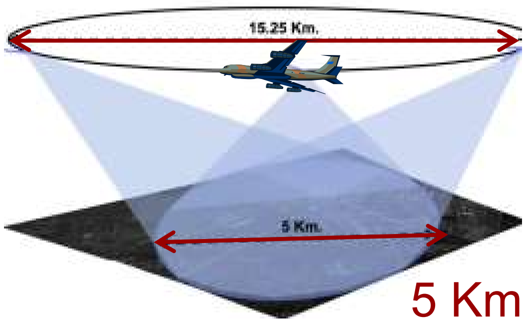




Jackson & RLM: *2009*

AFRL Gotcha Radar

15.25 Km



5 Km

Data Storage:

90 G samples/circle

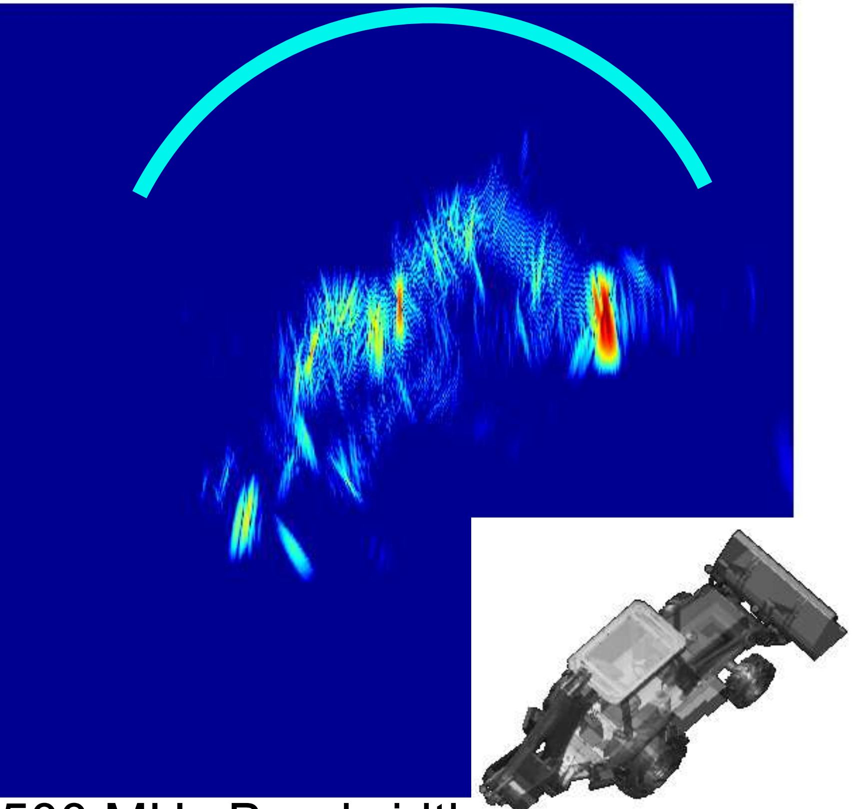
Image formation:

45 Tflops/sec

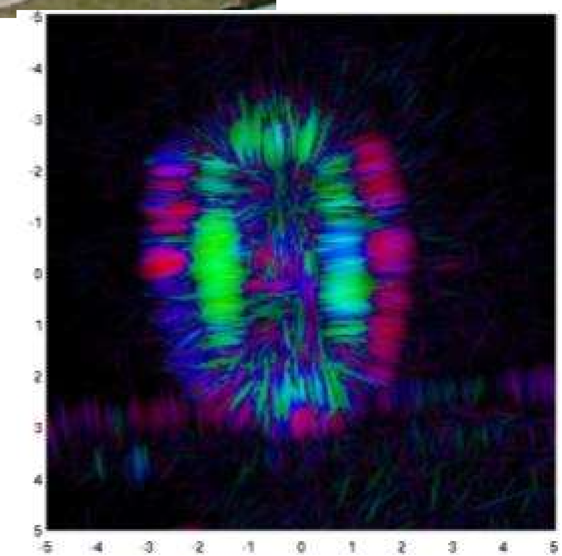
Communications:

190 M samples/sec

Coherent wide-angle SAR Images

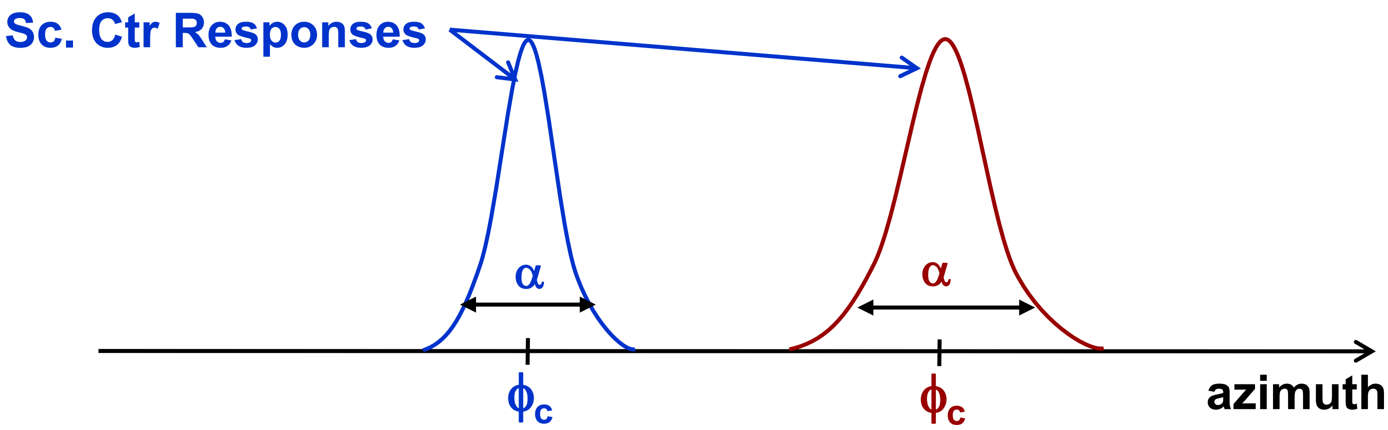


500 MHz Bandwidth
110 degrees az



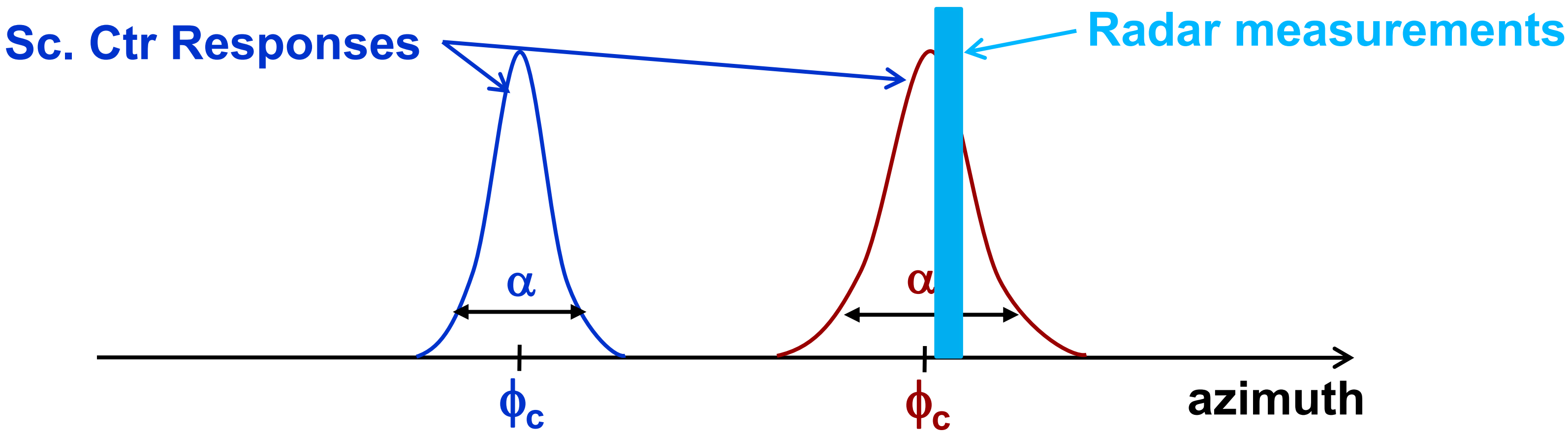
Coherent wide-angle image is not well-matched to limited persistence scattering behavior

Wide-Angle Data Collections

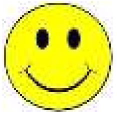


- Most backscatter does NOT behave like a point scatterer over wide angles
- Most scattering centers have limited response persistence
  - 20° or less at X-band [Dudgeon et al, 1994]
- Standard imaging is not statistically (close to) optimal

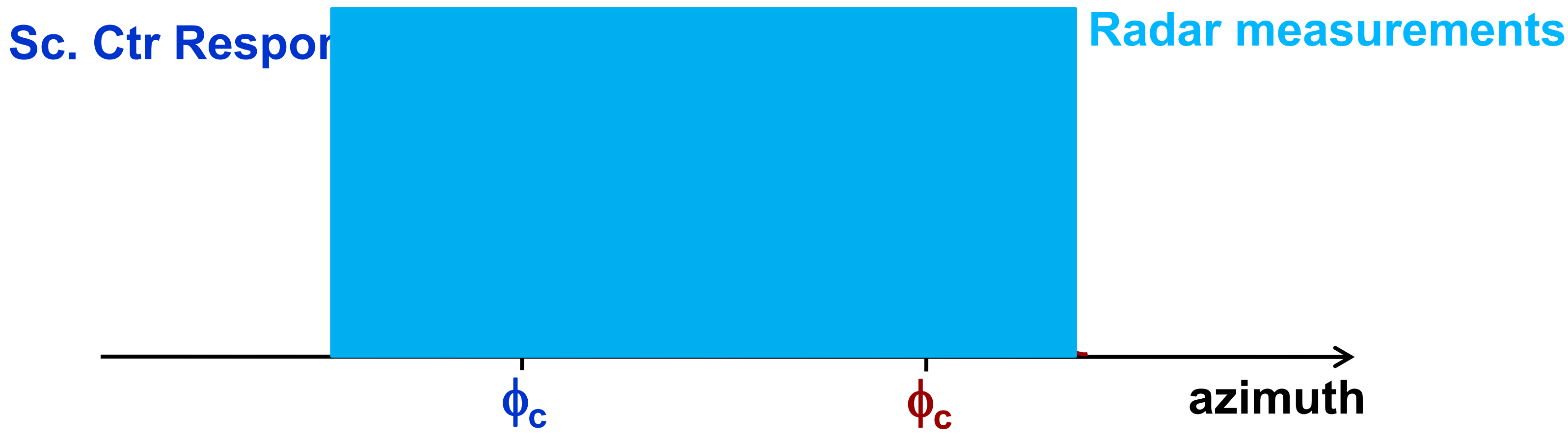
Wide-Angle Data Collections



When the radar measurement extent is ≤ scattering persistence, the isotropic assumption is ~satisfied, and tomographic imaging is ~a matched filter.



Wide-Angle Data Collections



For wide-angle measurements the isotropic scattering assumption breaks down.

– Tomography is no longer a matched filter 

Scattering Aspect Dependence

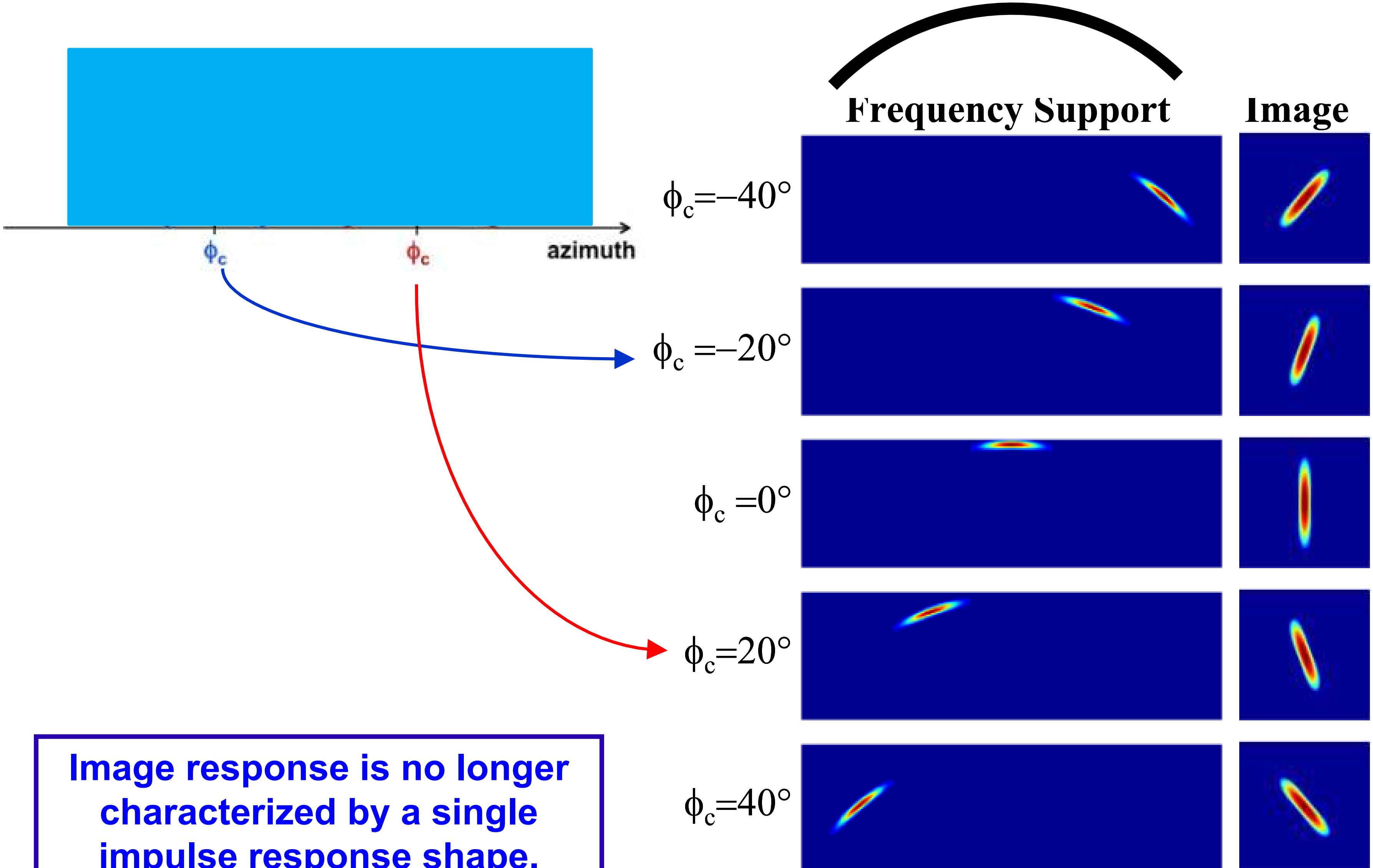
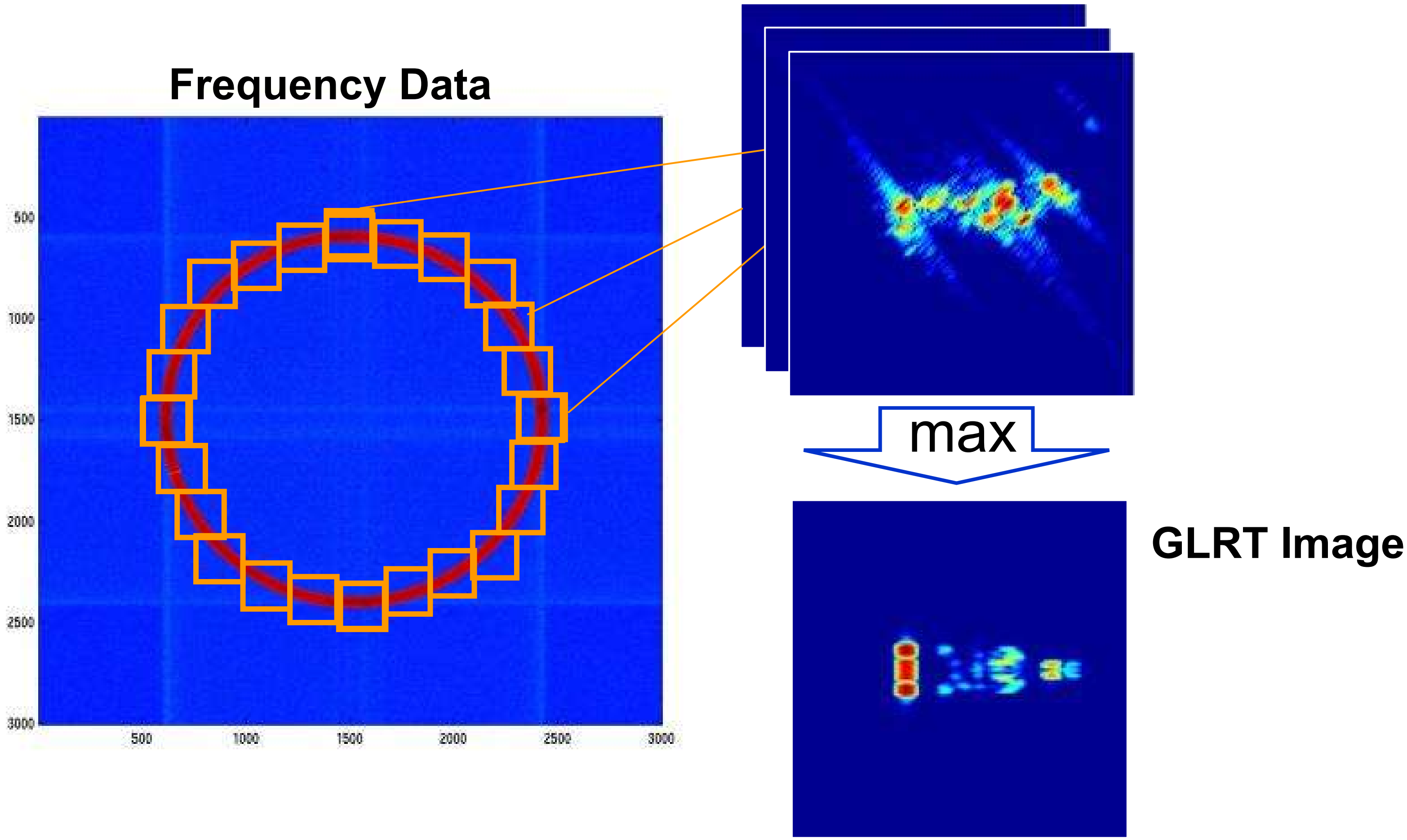


Image response is no longer characterized by a single impulse response shape.

GRLT Imaging



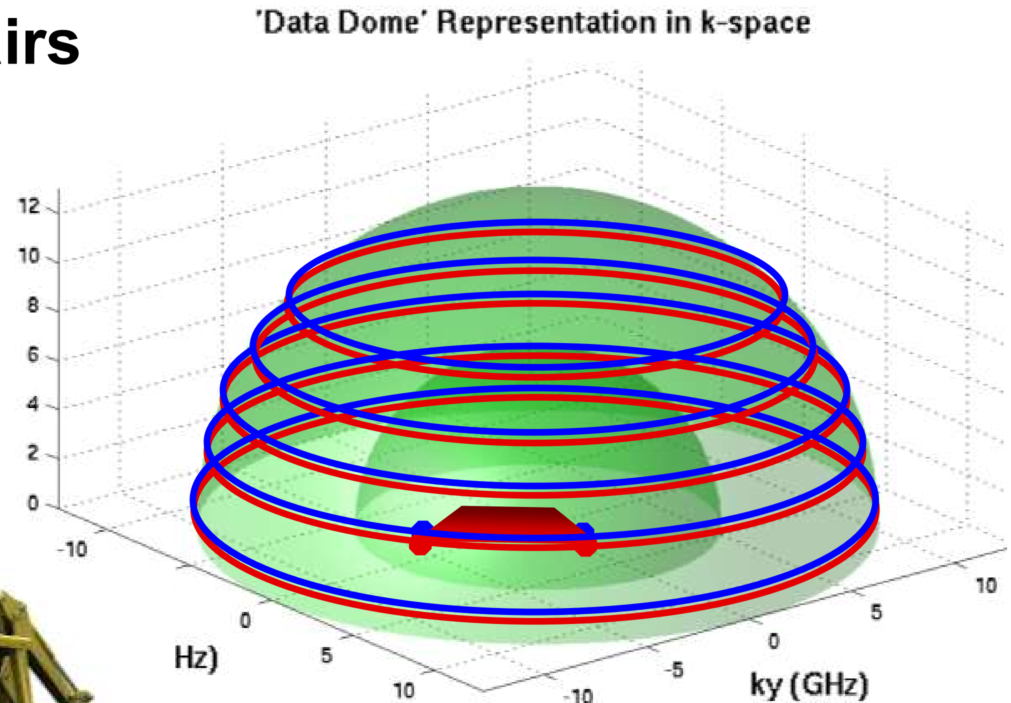
Generalizes Rossi+Willsky matched filter result to wide-angle imaging with limited-persistence scattering

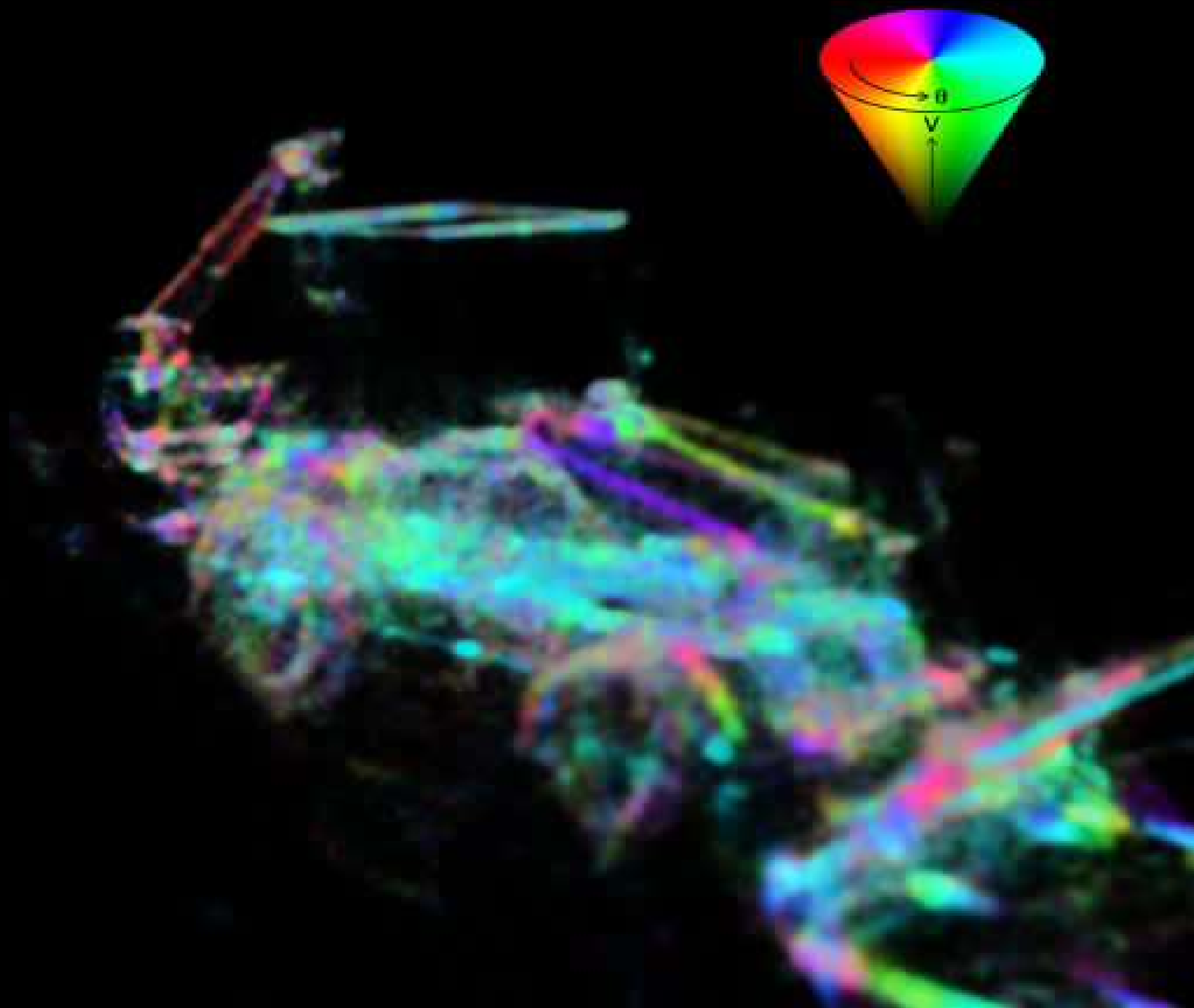
RLM, Potter, Cetin: *2004*

Sparse 3D Reconstruction, Take 1

Coherent IFSAR image pairs

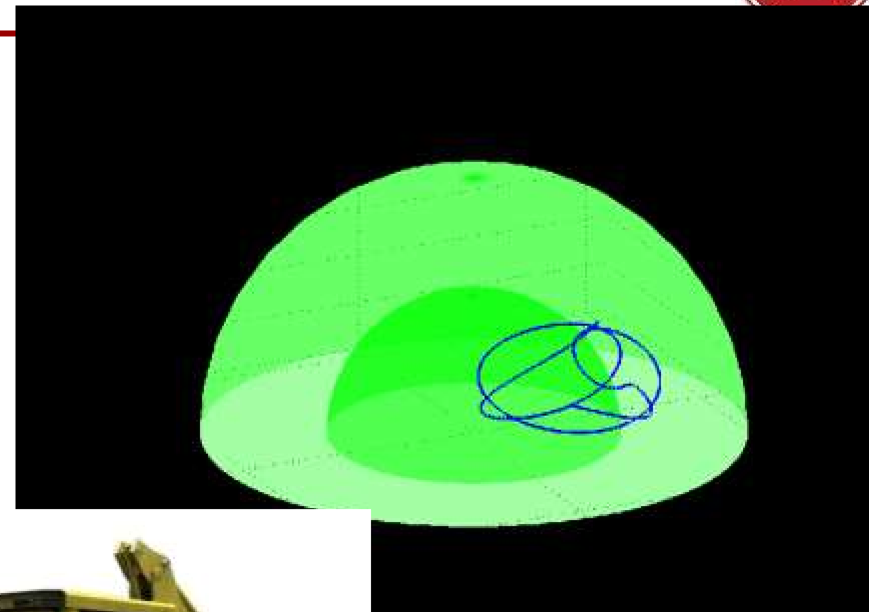
- 1.5”x1.5” resolution
  - 8-12 GHz
  - 24° aperture
- Every 5° elevation
  - $\Delta\theta = 0.05°$ elevation spacing
- 1296 total image pairs
- 2% of data used




θ
V

Sparse 3D Reconstruction: Take 2

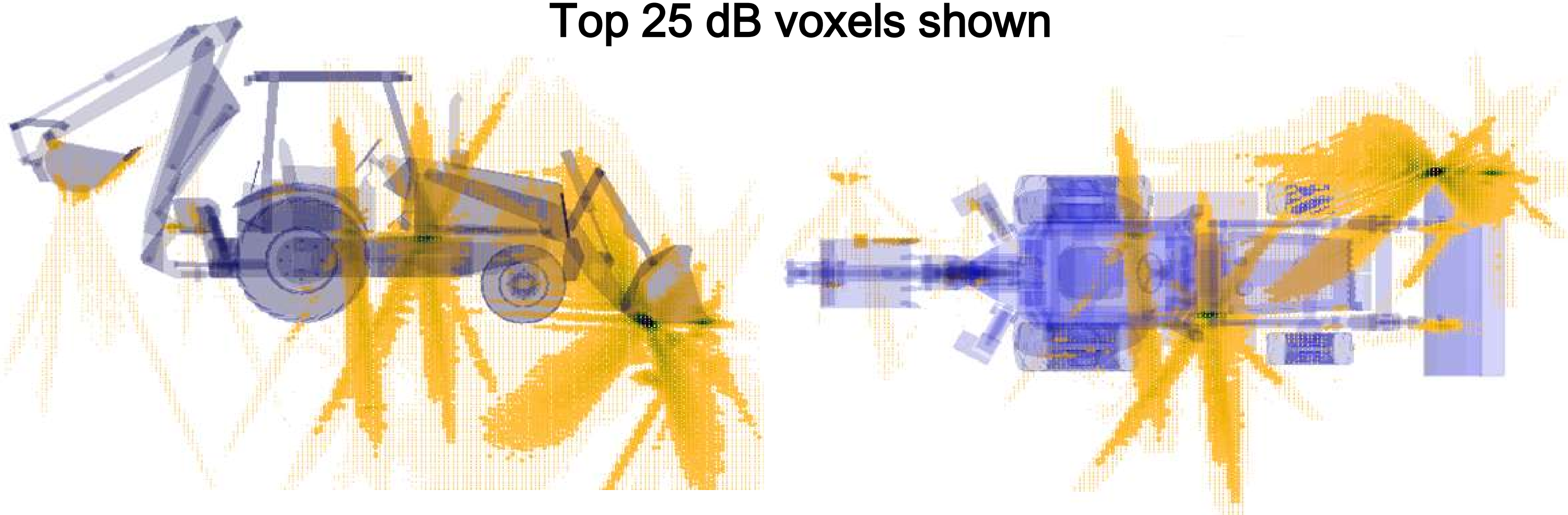
- 3D radar reconstruction necessarily will use (very) sparse measurements
- Is the radar reconstruction sufficiently sparse to overcome m sparsity?



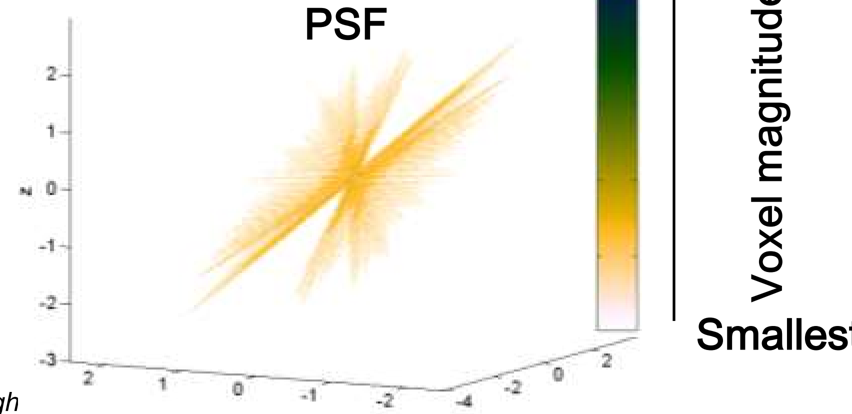
AFRL Backhoe Data Dome, with sparse "squiggle path" shown

Squiggle Path 3D Tomographic Reconstruction

Top 25 dB voxels shown

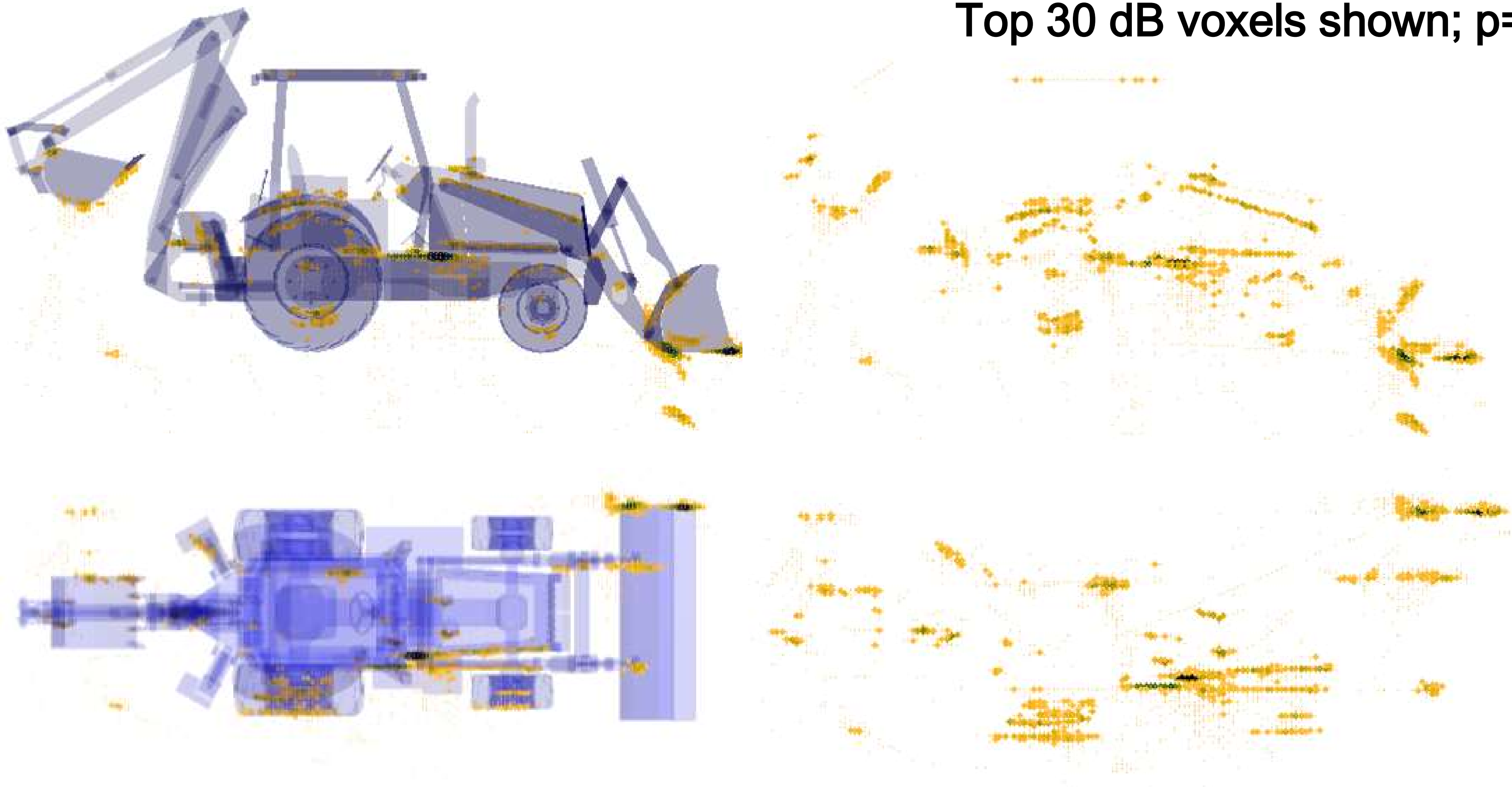


Squiggle PSF



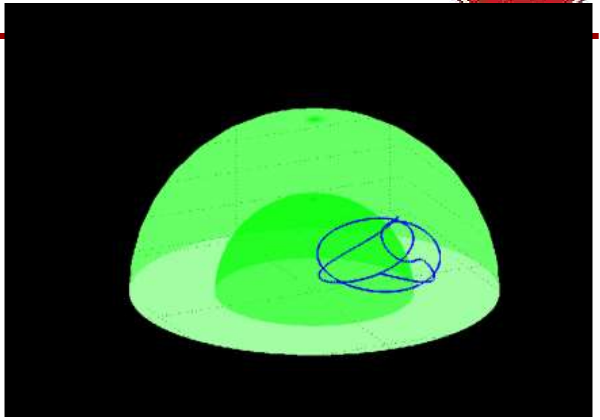
Squiggle Path Collection: l_p Regularized LS Reconstruction

Top 30 dB voxels shown; p=1

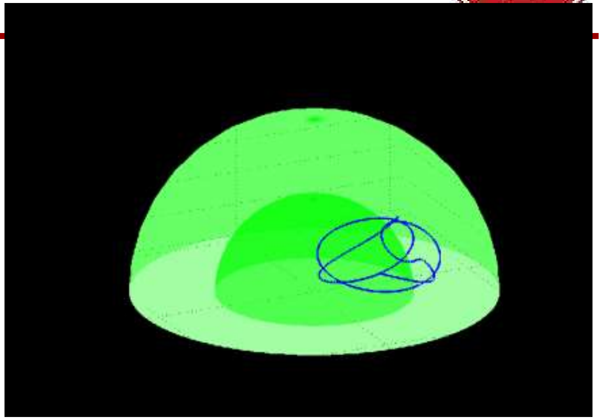


Austin, Ertin, RLM, 2011

Backhoe Squiggle Image



Backhoe Squiggle Image

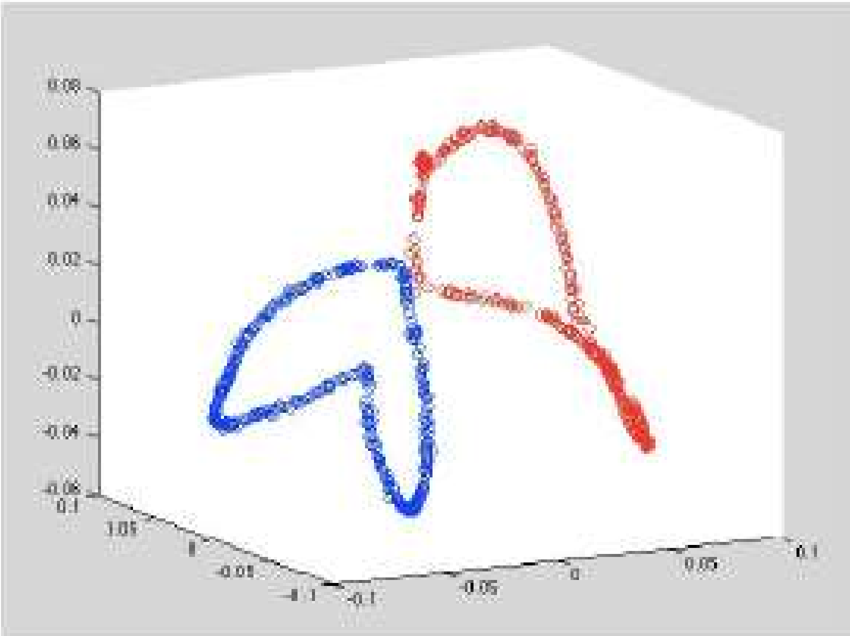


Gotcha l_p Reconstructions: Camry

- AFRL Gotcha Radar
- X-band circular SAR
- 500MHz bandwidth
- Public data releases

Austin, Ertin, RLM, 2011

Vehicle Classification; Attributed Point Sets



Camry:

Maxima:

Using standard feature classifiers, >95% correct classification is obtained for 10-class GOTCHA vehicle set using 500 MHz X-band circular SAR

Dungan and Potter, 2011

Newer Directions 1: Probabilistic

- Develop a Bayesian approach to Sparse Modeling
  - Output are full posterior distributions
    - Belief propagation using probalistic factor analysis
    - Robust co-estimation of ‘tuning’ parameters
  - Computation is comparable to CS

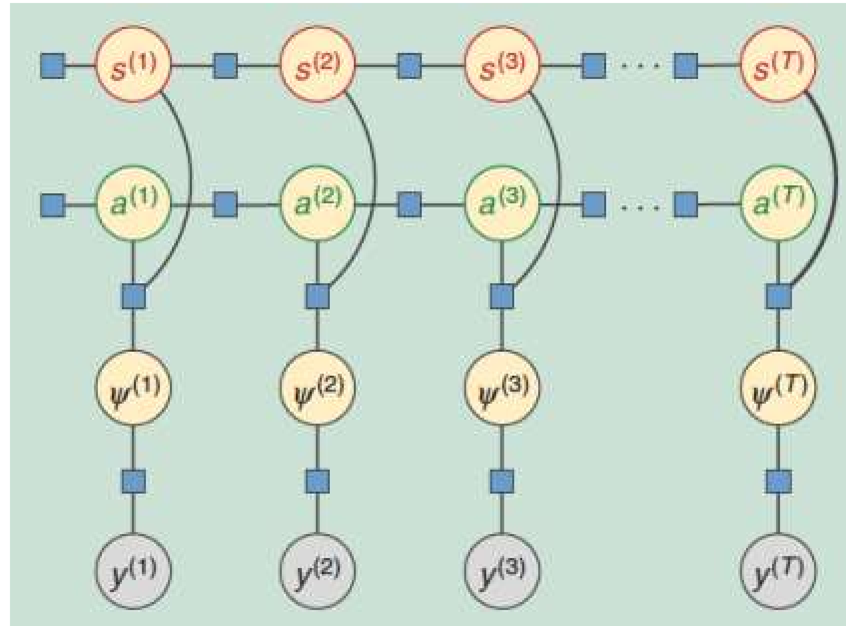
Modeling Azimuth Dependence

0 or 1

Pixel ampl.

Forward model

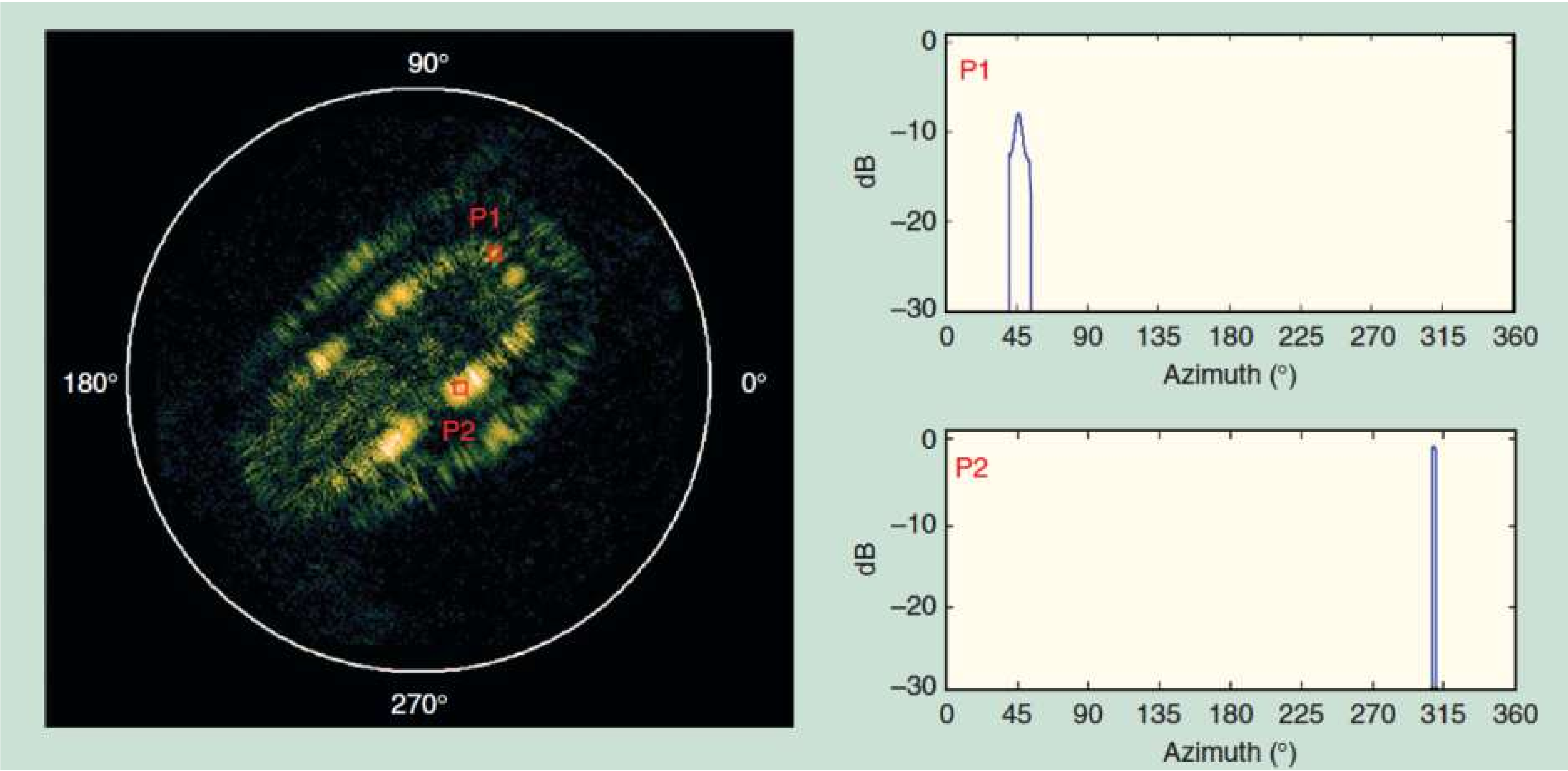
Phase History msmts



- **Develop a Bayesian approach to Sparse Modeling**
  - **Temporal (=azimuth) dependence model on aspect amplitude**
  - **Estimate of pdf for each variable**

From: J. Ash, E. Ertin, L. Potter, E. Zelnio, "Wide Angle Synthetic Aperture Radar," IEEE Signal Processing Magazine, **31**, 4, July 2014.

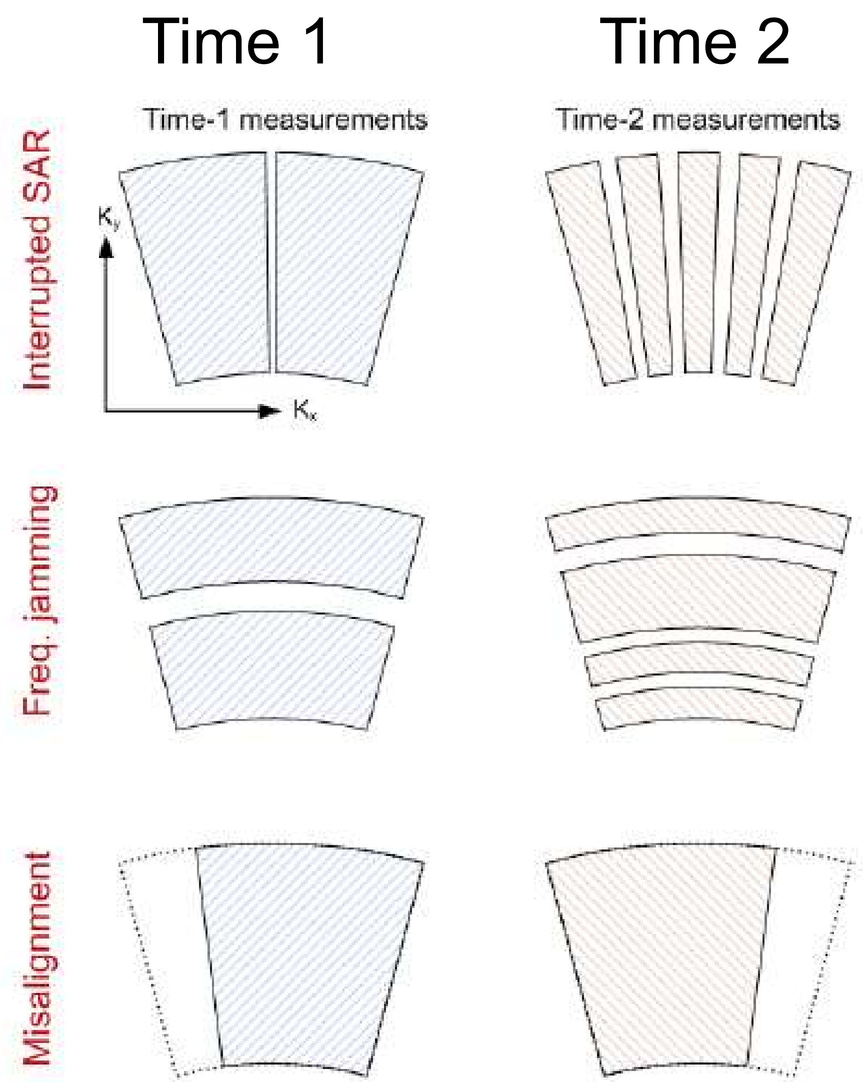
Azimuth Dependence Example



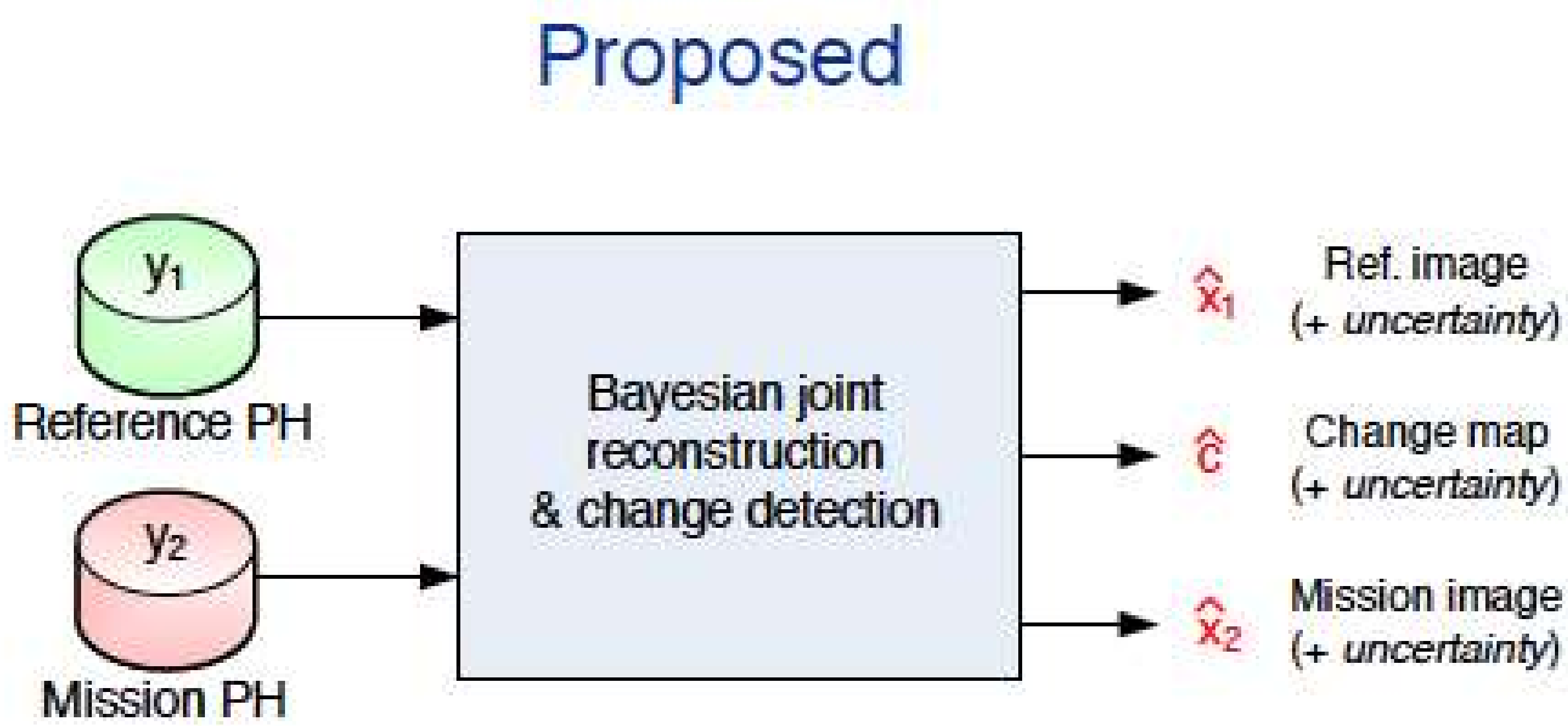
From: J. Ash, E. Ertin, L. Potter, E. Zelnio, "Wide Angle Synthetic Aperture Radar," IEEE Signal Processing Magazine, **31**, 4, July 2014.

Newer Directions 2: Change Detection

- Objective: Robust SAR change detection
  - under mixed sampling geometries
  - Interrupted apertures
  - Frequency jamming
  - Pass-to-pass misalignment



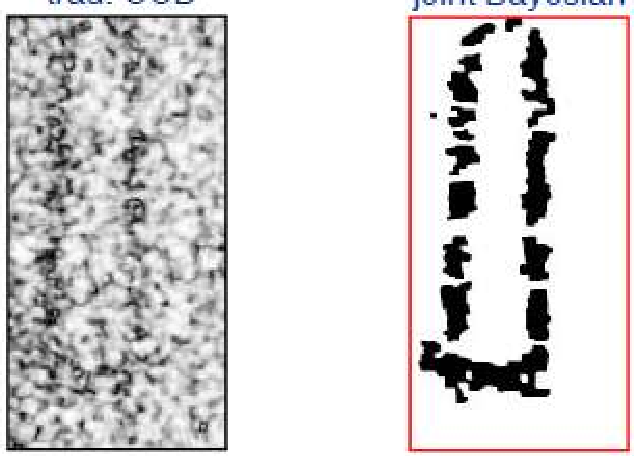
Newer Directions 2: Change Detection



Presented at: the 2014 Sensor Signal Processing for Defence Conference, Edinburgh, Scotland

Newer Directions 2: Change Detection

Discard 5% of Time1 data, 58% of Time2 data:

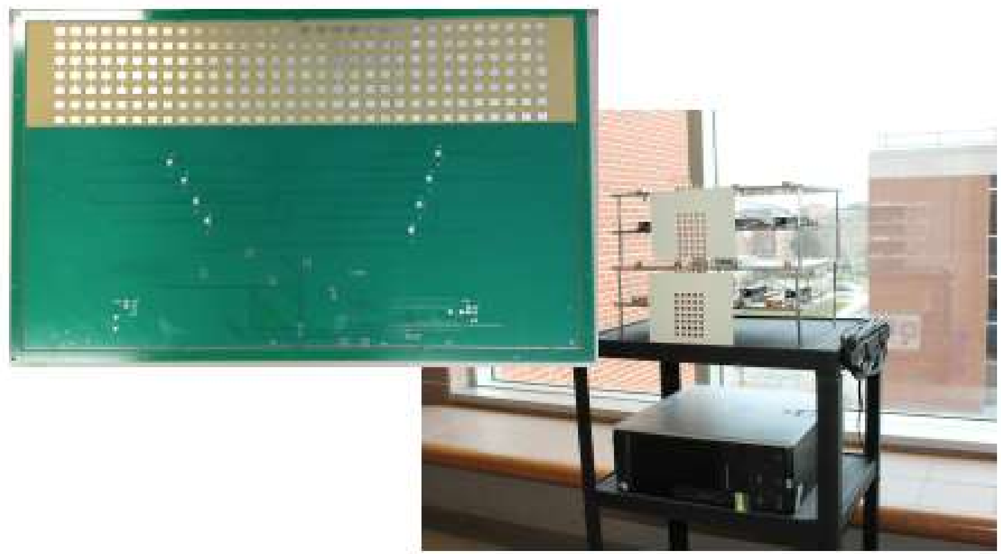


From: J. Ash, “A unifying perspective of coherent and non-coherent change detection,” Proc. SPIE. **9093**, Algorithms for Synthetic Aperture Radar Imagery XXI, 909309, June 2014.

Newer Directions 3: Low-Cost Hardware

- Distributed radar testbed consisting of 14 Micro SDRs.
  - Mobile form-factor, lightweight, fully digital programmable
- Colocated MIMO Radar system with 4 TX And 4 RX channels
  - airborne collection emulation using 32 TX and 32 RX antenna array
- Stand-alone, high-performance stationary infrastructure

Micro SDR



MIMO Radar System

Prof. Emre Ertin, ertin.1@osu.edu

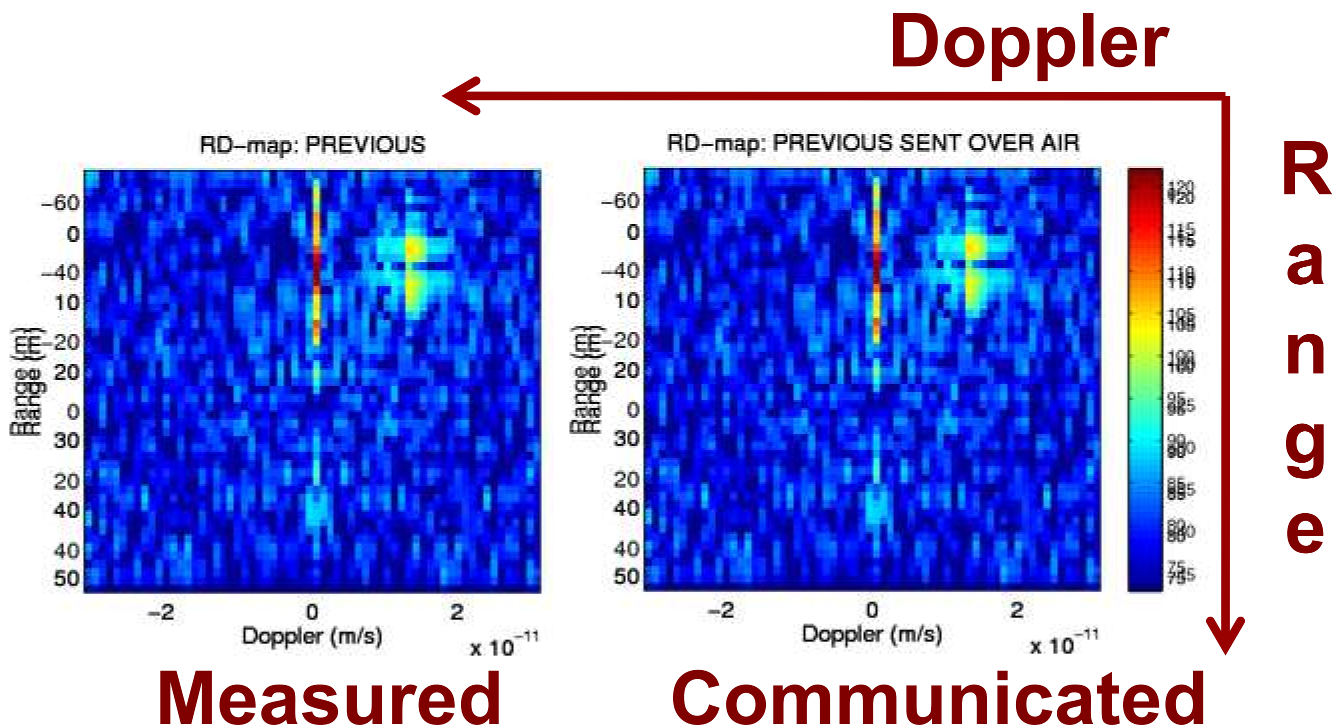
Newer Directions 3: Low-Cost Hardware

- 250 MHz Signal Bandwidth (60 cm resolution)
  - • Dual 250 MS/sec 14 bit A/D
  - • Dual 1 GS/sec oversampling 16 bit D/A
- Embedded Virtex-6 LX240T FPGA
- 215 mm (W) x 96 mm (H) x 290 mm (D)
- Custom X-Band RF-Frontend with switchable 4TX and 4RX Antenna Matrix
  - ability to chain for multiple units

Prof. Emre Ertin, ertin.1@osu.edu

Joint Sensing-Comm Experiment

- Self-adaptive joint radar/communication system
  - PN transmit signal waveform
- Measured and communicated range-Doppler maps
  - n^{th} range-Doppler map used to adapt $(n+1)^{st}$ waveform set.



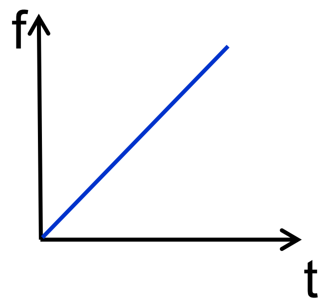
Rossler, Ertin, RLM: *2011*

Newer Directions 4: Transmit Design

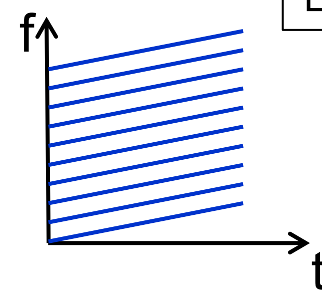
- Transmit signal design can alter A coherence properties
- Ex: 10 targets; 2 tx designs; 10:1 basis pursuit undersampling

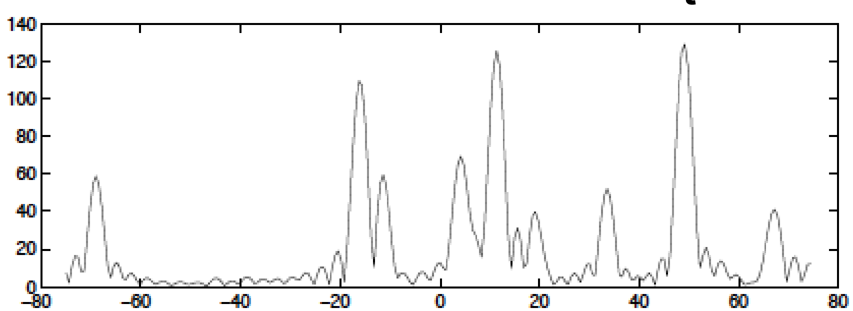
Ertin, SIAM 2012

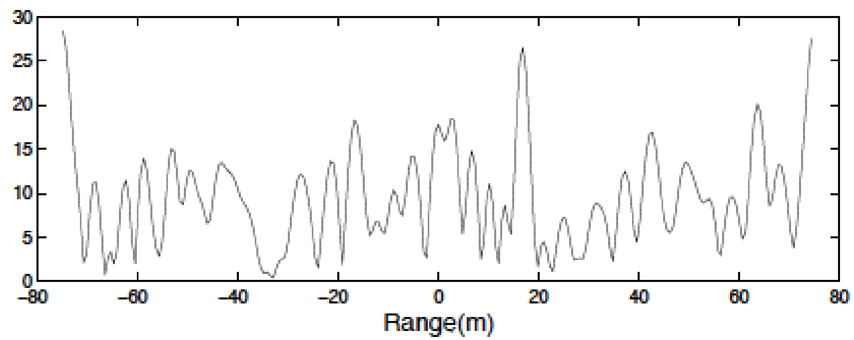
1 chirp

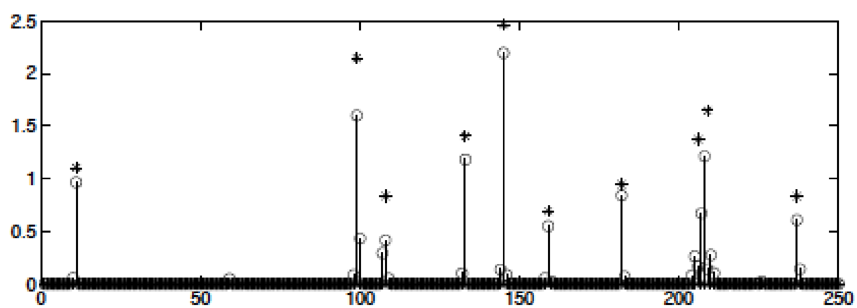


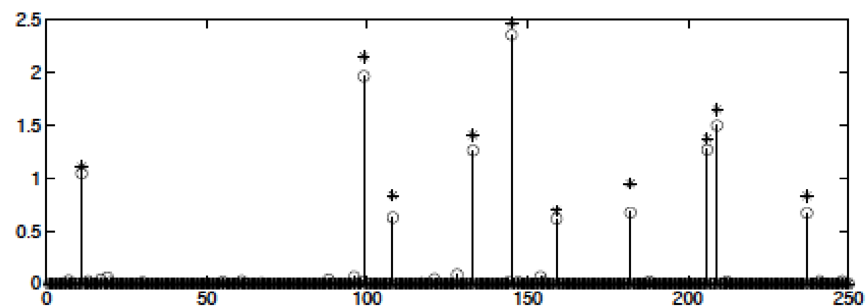
15 chirps





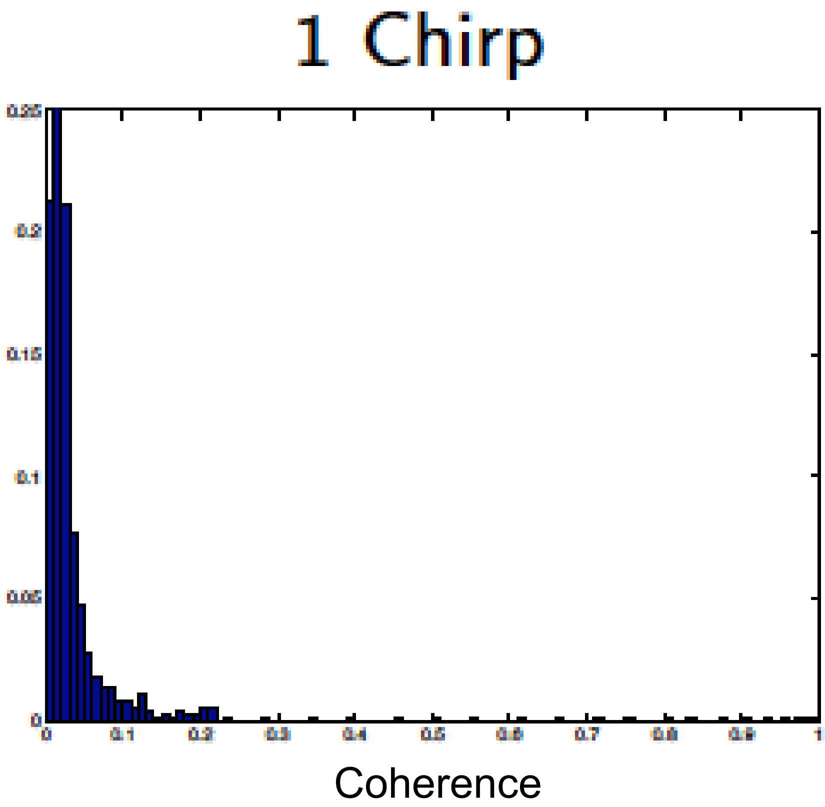


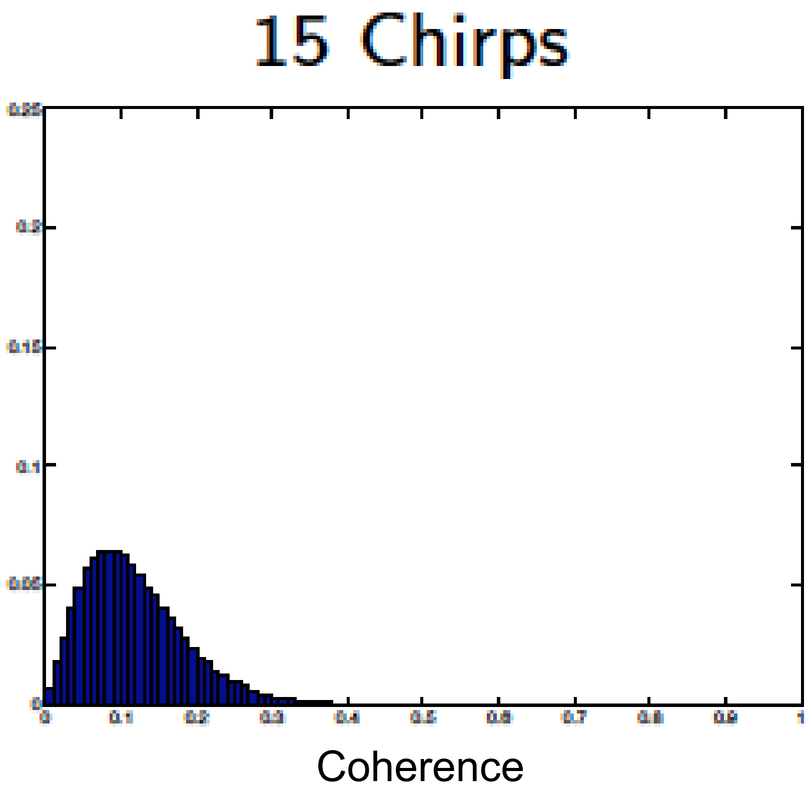




Transmit Designs for Coherence

Histogram of $A^H A$ magnitudes





Newer Direction 5: Relating CS to ML/MDL

- Can we related Sparse Reconstruction to parameter estimation?

$$\hat{x} = \arg\min_{x} \|y - Ax\|_2^2 + \lambda \|x\|_p^p \quad p \leq 1$$

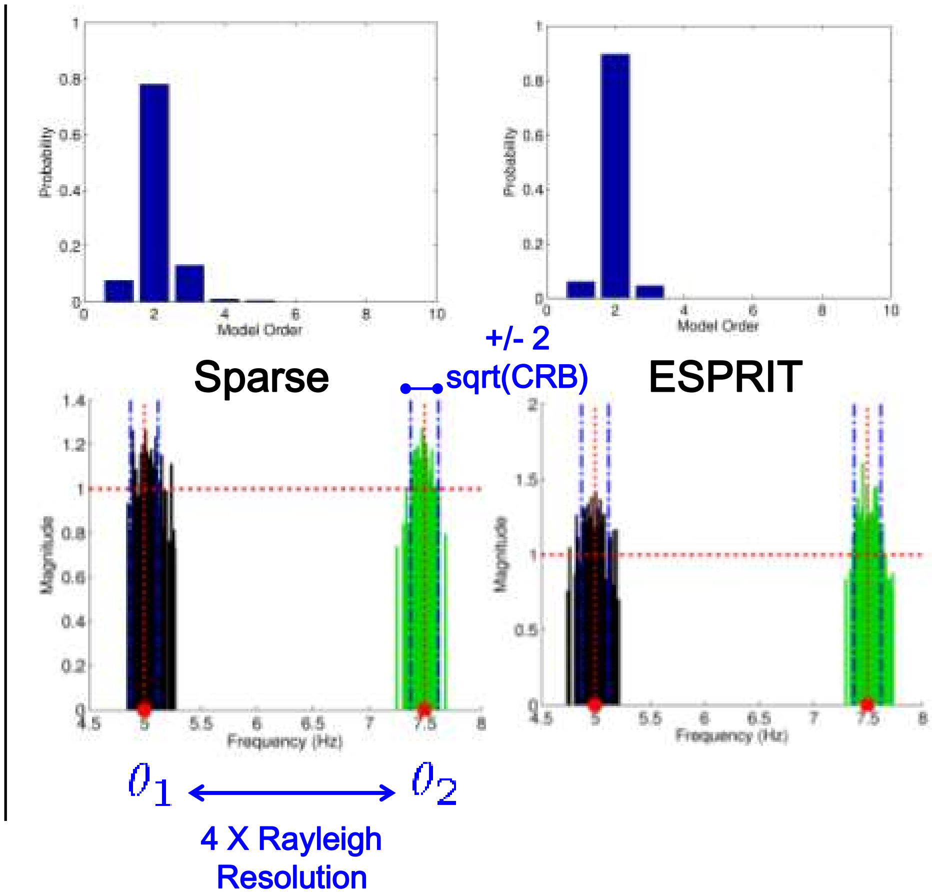
- MDL selection given by:

$$\lambda^* = \arg\min_{\lambda} J(\hat{x})$$

$$J(x) = \|y - Ax\|^2 + \frac{n_c \ln(N)}{2} \|x\|_0$$

Well-Separated Sinusoids

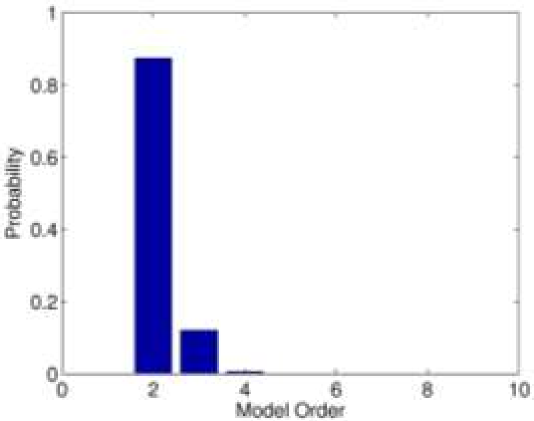
0 dB SNR

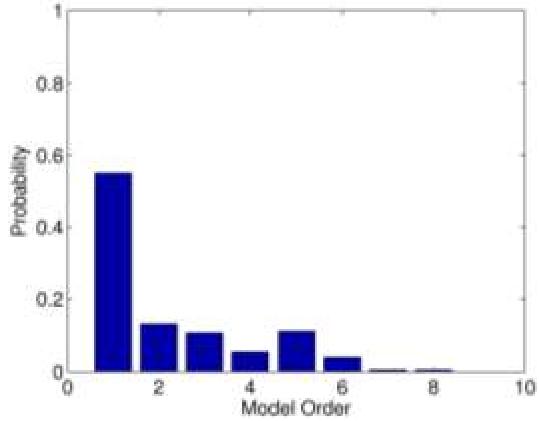


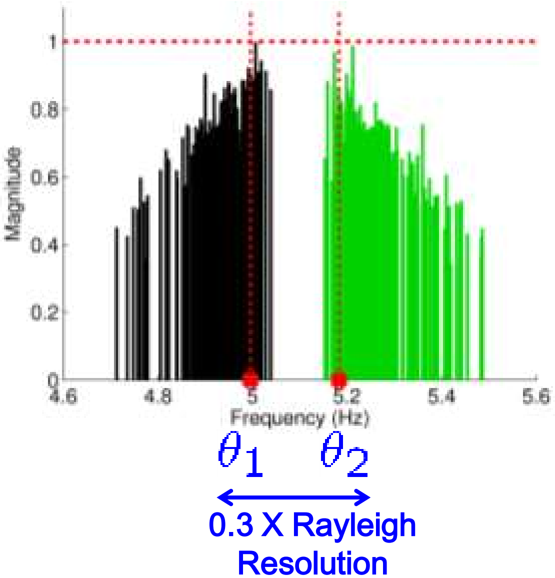
$\sqrt{\text{CRB } f_1} + \sqrt{\text{CRB } f_2}$	RMSE f_1 + RMSE f_2	
	Sparse	ESPRIT
0.1236	0.1642	0.1642

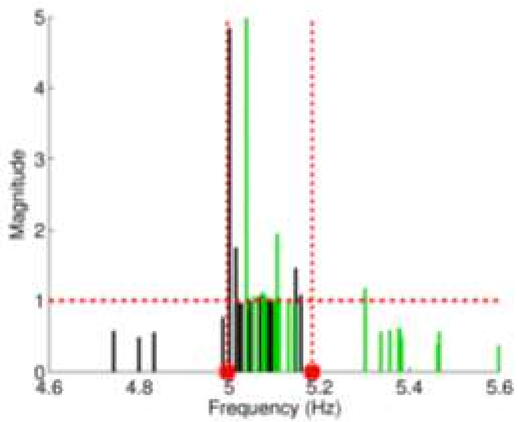
Closely-Spaced Sinusoids (Superresolution)

<u>10 dB SNR</u>









$\sqrt{\text{CRB } f_1} + \sqrt{\text{CRB } f_2}$	RMSE f_1 + RMSE f_2	
	Sparse	ESPRIT
0.1772	0.1951	1.6572

Presented at: the 2014 Sensor Signal Processing for Defence Conference, Edinburgh, Scotland

Closing Points

- Advances in sampling and digital processing are moving radar systems more firmly in the digital realm.
  - Much broader set of signaling and waveform adaptation possibilities
- Persistence and wide-angle sensing motivate rethinking the models and algorithms for radar processing.
  - Sparse nonparametric and parametric solutions
  - New opportunities for using the time dimension
- A rich collaboration across diverse research communities are steadily producing algorithms and enabling hardware proving effective on real-world radar challenges.

Thank you!