Outage Analysis of OFDM based AF Cooperative Systems in Selection Combining Receiver over Nakagami-m fading channels with Nonlinear Power Amplifier

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Abstract

In this paper, we investigate the performance of orthogonal frequency division multiplexing (OFDM) based amplify-and-forward (AF) cooperative relaying systems over independent and identically distributed (i.i.d) Nakagami-m fading channels. Specifically, we derive a closed-form expression for lower bound of outage probability by using selection combining (SC) scheme at the receiver. A nonlinear power amplifier (PA) is considered at the relay which introduces the nonlinear distortions. We present simulation and numerical results to validate the theoretical analysis, and demonstrates the impact of nonlinearity of PA parameters over outage probability for different values of threshold signal-to-noise ratio (SNR) at various mean SNR levels.

Introduction

- In today's world, the demand from wireless communications to provide high bit rate and coverage in hostile environments/terrains is of prime importance for defence and natural security.
- Cooperative communication systems have ability to enhance the coverage and to increase the capacity of wireless communication link by exploiting the spatial diversity without the need of multiple antennas [1].
- OFDM is the key element of many wireless communication standards and plays an important role in broadband communications due to its ability to mitigate the intersymbol interference (ISI), intercarrier interference (ICI) and provide high spectral efficiency [2].
- High peak-to-average power ratio (PAPR) is characterised as one of the major drawback of the signals at the output of the OFDM block [3]. If a high input back-off is not used, a high PAPR level is the main reason for introducing nonlinearities in the received signals, which come from the saturated mode of PAs [4].
- After intensive literature survey, authors' propose a closed-form expression for the lower bound of outage probability for a more general distribution model as frequency selective Nakagami-m fading channels considering nonlinear PA at the relay and using best relay selection scheme at receiver.

System Model

We consider a cooperative OFDM system model with AF relaying scenario which is shown in Figure 1. This system model consists of source node (S) that communicates with the destination node (D) through the direct link $S \to D$ and an indirect link $S \to R \to D$ where R denotes the relay. Source, relay and destination node, each consists of one transmitting and one receiving antenna. All communication between nodes operate in half-duplex mode and all the nodes are synchronised at symbol level. All the wireless channels are assumed as independent frequency selective Nakagami-m fading channels and the length of the OFDM cyclic prefix is higher than or equal to the maximum multipath delay of all the wireless channels. All the subcarriers of the source and relay consists of same transmission power P_s and P_r, respectively. A nonlinear PA at the relay node is considered that is modeled by a soft clipper [5]. At the receiver selection combining is considered, thus, end-to-end SNR is given as

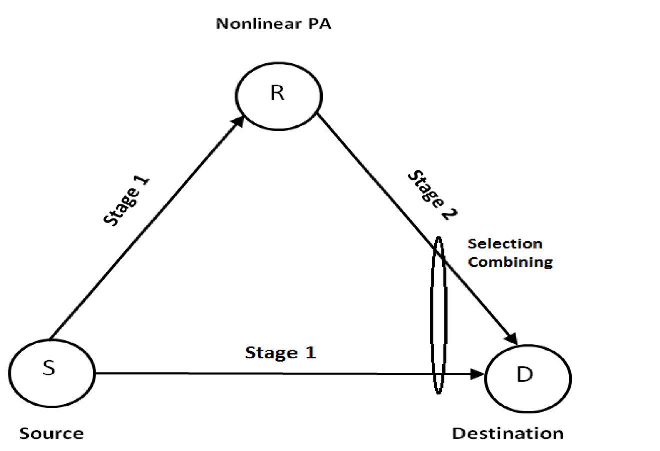

Figure 1: Cooperative system model

$$\gamma_n^{SC} = \max(\gamma_n^{SD}, \gamma_n^{SRD}), \quad 1 \leq n \leq N \tag{1}$$

where N represents the number of subcarriers, γ_n^{SD} is the instantaneous SNR of the n^{th} subcarrier of $S \to D$ link, γ_n^{SRD} is the SNR of $S \to R \to D$ link as given in [6] as

$$\gamma_n^{SRD} = \frac{\gamma_n^{SR}\gamma_n^{RD}\gamma^{PA}}{\gamma_n^{SR}\gamma_n^{RD} + \gamma_n^{SR}\gamma^{PA} + \gamma_n^{RD}\gamma^{PA} + \gamma_n^{RD} + \gamma^{PA}} \tag{2}$$

where γ_n^{SR} and γ_n^{RD} are the instantaneous SNRs of the n^{th} subcarrier of $S \to R$ and $R \to D$ links, respectively. Also $\gamma^{PA} = |K_0^R|^2 P_r/\sigma_{d^R}^2$ is the instantaneous SNR at the output of the PA, where K_0^R and $\sigma_{d^R}^2$ are the complex-valued constant gain and the variance of frequency domain nonlinear distortion [6]. Further, the approximated upper bound value of instantaneous SNR for $S \to R \to D$ link given in (2) can be expressed as [6]

$$\gamma_n^{SRD} \leq \min(\gamma_n^{SR}, \gamma^{PA}, \gamma_n^{RD}) \tag{3}$$

Outage Probability Analysis

The outage probability for n^{th} subcarrier can be defined as the probability that the end-to-end SNR (γ_n^{SC}) falls below a predefined threshold SNR (γ_{th}). Thus,

$$P_n^{out,LB}(\gamma_{th}) = \Pr(\gamma_n^{SC} < \gamma_{th}) \tag{4}$$

where $\gamma_n^{SC} = \max(\gamma_n^{SD}, \gamma_n^{\min})$, is the upper bound SNR at the SC receiver with $\gamma_n^{\min} = \min(\gamma_n^{SR}, \gamma^{PA}, \gamma_n^{RD})$. Substituting value of γ_n^{SC} in (4), we get

$$\begin{aligned} P_n^{out,LB}(\gamma_{th}) &= \Pr\{\max(\gamma_n^{SD}, \gamma_n^{\min}) < \gamma_{th}\} \\ &= F_{\gamma_n^{SD}}(\gamma_{th}) F_{\gamma_n^{\min}}(\gamma_{th}) \end{aligned} \tag{5}$$

where $F_X(.)$ denote the cumulative distribution function (CDF) of the corresponding random variable. After some mathematical calculations, $F_{\gamma_n^{\min}}(\gamma_{th})$ can be obtained as

$$F_{\gamma_n^{\min}}(\gamma_{th}) = u(\gamma_{th}) - \frac{\Gamma(m, \frac{m\gamma_{th}}{\bar{\gamma}^{SR}})}{\Gamma(m)} \times \frac{\Gamma(m, \frac{m\gamma_{th}}{\bar{\gamma}^{RD}})}{\Gamma(m)} u(\gamma_{th}) u(\bar{\gamma}^{PA} - \gamma_{th}) \tag{6}$$

Substituting the CDF of $S \to D$ link and value of $F_{\gamma_n^{\min}}(\gamma_{th})$ from (6) into (5) and further some mathematical manipulations, we can derived the tight lower bound of outage probability for OFDM based AF cooperative system as

$$\begin{aligned} P_n^{out,LB}(\gamma_{th}) = \Bigg[1 - \frac{\Gamma\left(m, \frac{m\gamma_{th}}{\bar{\gamma}^{sd}}\right)}{\Gamma(m)} - \frac{\Gamma(m, \frac{m\gamma_{th}}{\bar{\gamma}^{sr}})}{\Gamma(m)} \frac{\Gamma(m, \frac{m\gamma_{th}}{\bar{\gamma}^{rd}})}{\Gamma(m)} u(\bar{\gamma}^{PA} - \gamma_{th}) + \\ \frac{\Gamma\left(m, \frac{m\gamma_{th}}{\bar{\gamma}^{sd}}\right)}{\Gamma(m)} \frac{\Gamma(m, \frac{m\gamma_{th}}{\bar{\gamma}^{sr}})}{\Gamma(m)} \frac{\Gamma(m, \frac{m\gamma_{th}}{\bar{\gamma}^{rd}})}{\Gamma(m)} u(\bar{\gamma}^{PA} - \gamma_{th}) \Bigg] u(\gamma_{th}) \end{aligned} \tag{7}$$

Numerical and Simulation Results

An OFDM based AF cooperative system is considered, where we use QPSK modulated signals, $N = 64$ subcarriers, cyclic prefix length is 16 and frequency selective Nakagami-m fading channels with 16 independent taps. Furthermore, a nonlinear PA at relay is modeled by a soft clipper (soft limiter) with $A_{sat} = 1$ and $P_s = P_r = 1$ which leads $\bar{\gamma}^{PA} = 12$ dB. All channels are considered as identical and independent with equal noise variances, hence $\bar{\gamma}^{SR} = \bar{\gamma}^{RD} = \bar{\gamma}^{SD} = \bar{\gamma}$.

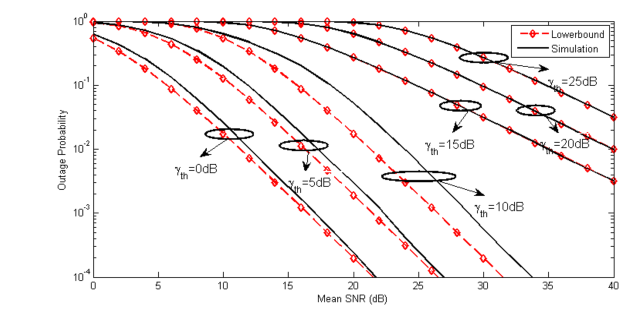

Figure 2: Comparision between lower bound and simulation results of outage probability versus mean SNR over Nakagami-m channel with $m = 1$ (Rayleigh) and various values of γ_{th}

Figure 2 shows the comparision between lower bound and simulation results of outage probability versus mean SNR, $\bar{\gamma}$ over Nakagami-m fading channel with fading parameter, $m = 1$ (Rayleigh fading). The outage probability being obtained for various values of threshold SNR, γ_{th}. It can be observed that the lower bound and simulation results are very close to each other (gap less than 1 dB) for most of the cases. When $\gamma_{th} = 10$dB, gap between the curves is increased due to the reason that for equal values of γ_{th} and $\bar{\gamma}^{PA}$ does not give good approximation [4]. By the slopes of the curves, we can observe that the system diversity gain is higher when $\gamma_{th} < \bar{\gamma}^{PA}$. This is due to the fact the nonlinear distortion is more significant at high SNRs than low SNRs.

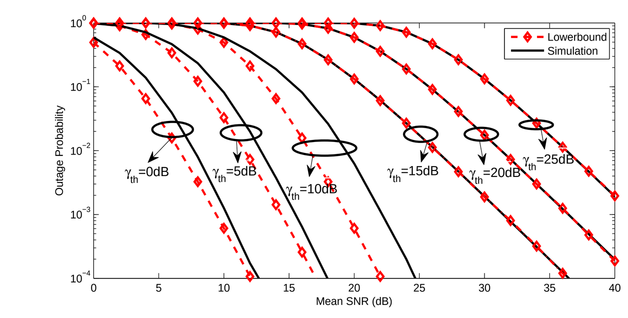

Figure 3: Comparision between lower bound and simulation results of outage probability versus mean SNR over Nakagami-m channel with $m = 2$ and various values of γ_{th}

Figure 3 shows the comparision between lower bound and simulation results of outage probability versus mean SNR, $\bar{\gamma}$ over Nakagami-m fading channel with fading parameter, $m = 2$. These curves show similar behaviour as in Figure 2, but the outage probability is further improved, as expected. Hence, conclusions for Figure 2 (when $m = 1$) is also applicable for Figure 3 (when $m = 2$).

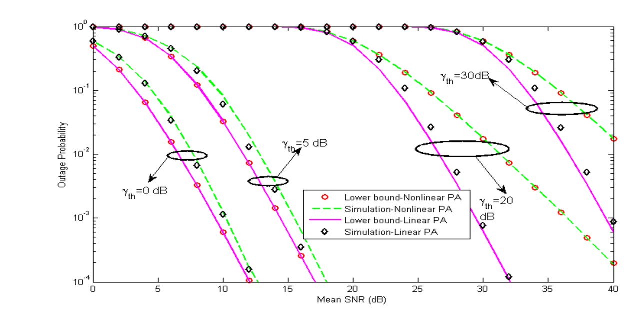

Figure 4: Comparision between lower bound and simulation results of outage probability versus mean SNR over Nakagami-m channel with $m = 2$, various values of γ_{th} and linear and nonlinear PAs

Figure 4 describes the comparision between lower bound and simulation results of outage probability versus mean SNR, $\bar{\gamma}$ over Nakagami-m fading channel with fading parameter, $m = 2$, considering linear and nonlinear PAs, for various values of threshold SNR. It can be observed that for $\gamma_{th} < \bar{\gamma}^{PA}$, nonlinear PA behaves like linear PA, however when $\gamma_{th} > \bar{\gamma}^{PA}$, the impact of nonlinearity is increased over outage probability. The difference between the outage probability curves of linear and nonlinear PAs is further increased by increasing the value of γ_{th}. Hence, it can be conclude that the nonlinearity of PA affects the system outage probability only for high values of γ_{th}.

Conclusions

- We have analysed the performance of orthogonal frequency division multiplexing (OFDM) based amplify-and-forward (AF) cooperative relaying scheme over i.i.d Nakagami-m fading channels.
- We derived a closed-form expression for lower bound of outage probability with SC receiver.
- The comparison of numerical and simulation results demonstrated that the derived expression of outage probability has good approximation in most of the cases, specially at low and high SNRs.
- It can also be observed that PA nonlinearity degrades the outage probability performance only at high threshold SNRs.
- At low threshold SNR nonlinear PA behaves like linear PA.

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