Quadrature Filters for Underwater Passive Bearings-Only Target Tracking

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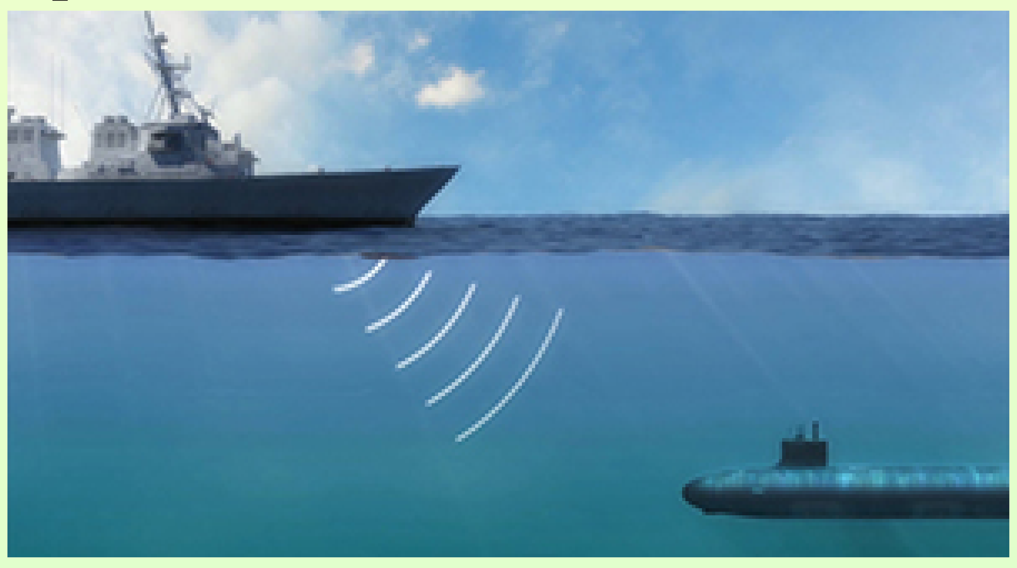
Abstract

A typical underwater passive bearings-only target tracking problem is solved using nonlinear filters namely cubature Kalman filter (CKF), Gauss-Hermite filter (GHF) and sparse-grid Gauss-Hermite filter (SGHF). The performance of the filters is compared in terms of estimation accuracy, track-loss count and computational time. Theoretical Cramer-Rao lower bound (CRLB) is used to determine the maximum achievable performance and to compare the error bounds of various filters used.

Introduction

- The probelm of bearings-only tracking finds it's application in many real-life scenarios like aircraft surveillance, underwater tracking *etc*.
- Objective is to find the kinematics of a moving target such as range, speed *etc*. from noise corrupted bearings-only measurements.
- Since measurements are obtained from a passive sonar mounted on a warship, it helps in concealing the identity of the host ship [1].



- The problem becomes challenging due to the nonlinear nature of the measurements and non-availability of optimal solution.
- For sub-optimal solutions, several nonlinear filters like the extended Kalman filter (EKF), unscented kalman filter(UKF), the cubature Kalman filter (CKF), the Gauss-Hermit filter(GHF), the sparse-grid Gauss-Hermite filter (SGHF) *etc* have been developed, where the intractable integrals are approximated numerically.
- In this work, the CKF, GHF [2] and SGHF [3] are used to solve this nonlinear tracking problem.

Problem Formulation

- The state vector denoting target dynamics can be defined as $\mathbf{x}_k^t = [x_k^t \; y_k^t \; \dot{x}_k^t \; \dot{y}_k^t]^T$ and observer state dynamics can be defined as $\mathbf{x}_k^o = [x_k^o \; y_k^o \; \dot{x}_k^o \; \dot{y}_k^o]^T$. Now, the relative state vector is $\mathbf{x}_k \triangleq \mathbf{x}_k^t - \mathbf{x}_k^o = [x_k \; y_k \; \dot{x}_k \; \dot{y}_k]^T$.
- The discrete time state equation can be expressed as [1]:
$$\mathbf{x}_k = F\mathbf{x}_{k-1} + v_{k-1} - U_{k-1,k}. \qquad (1)$$
- F is the state transition matrix and v_{k-1} is a zero mean Gaussian process noise vector with covariance matrix Q.
- $U_{k-1,k}$ carries the observer accelerations which is to be included in target dynamics.

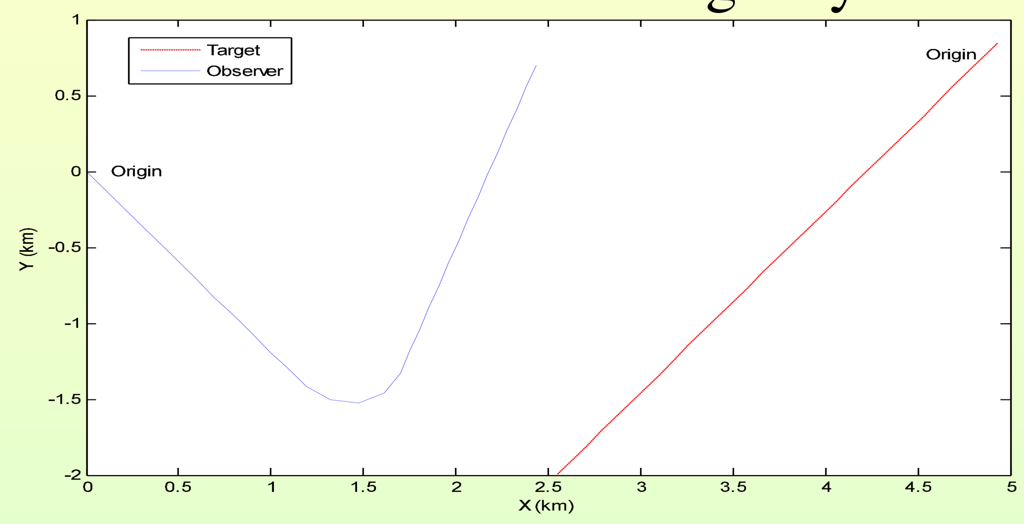


Figure 1: Target-Observer scenario

- The measurement equation can be represented as:
$$z_k = \gamma(\mathbf{x}_k) + \eta_k, \qquad (2)$$
where η_k is a zero mean Gaussian noise with standard deviation σ_θ. The true bearing measurements, with reference to true north are defined as, $\gamma(\mathbf{x}_k) = tan^{-1}(x_k/y_k)$.

Sparse-Grid Gauss-Hermite filter

- Using Smolyak rule, the integral of interest could be approximated as
$$I_{n,L}(f) \approx \sum_{q=L-n}^{L-1} (-1)^{L-1-q} C_{L-1-q}^{n-1} \sum_{\Xi \in \mathbf{N}_q^n} \sum_{q_{s_1} \in X_{l_1}} \dots \sum_{q_{s_n} \in X_{l_n}} f(q_{s_1}, \dots, q_{s_n}) w_{s_1} \dots w_{s_n} \quad (3)$$
where X_{l_j} is the set of single dimensional quadrature points corresponding to the univariate quadrature rule I_{l_j}, L is accuracy level, $[q_{s_1}, q_{s_2}, \dots, q_{s_n}]^T$ is a sparse-grid quadrature (SGQ) point, $N_q^n = \Xi : \sum_{j=1}^n l_j = n+q$ if $q \geq 0$ else $\varnothing$. $q_{s_j} \in X_{l_j}$ and w_{s_j} is the weight associated with q_{s_j}. The final set of SGQ points can be expressed as $X_{n,L} = \bigcup_{q=L-n}^{L-1} \bigcup_{\Xi \in \mathbf{N}_q^n} (X_{l_1} \otimes X_{l_2} \otimes \dots \otimes X_{l_n})$, where $\bigcup$ represents union.
- Fig 2, illustrates the construction of sparse-grid Gauss-Hermite points for a two dimensional system with third degree of accuracy level ($L = 3$).

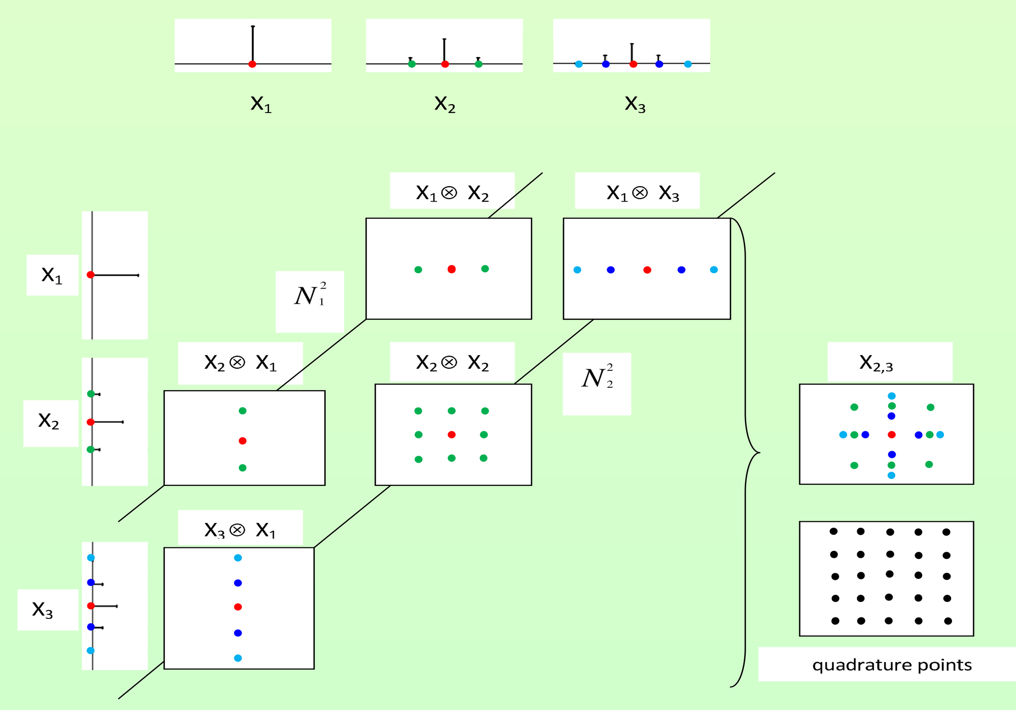


Figure 2: Multidimensional sparse-grid Gauss-Hermite points for $n = 2$, $L = 3$

Simulation results

- The filters were initialised using the method given in [1].
- For simulations, accuracy level of both GHF and SGHF was taken as 2.
- The parameters considered for tracking problem are mentioned in the table below.

Parameters	values
Initial range (r)	5 km
Target speed (s)	4 $knots$
Target course	-140^o
Observer speed	5 $knots$
Observer initial course	140^o
Observer final course	20^o
Observer manoeuver	$13^{th}to17^{th}$ min

- Sampling interval, $T = 1min$ and observation lasted for $28min$. A total of 500 monte carlo runs were done for comparing various filter performances as shown in table below.
- RMSE was calculated considering a track-loss condition, in which a track was defined divergent when the position error at any time index exceeds $5km$.

Filters	Track-loss	Compu. time
CKF	5.6%	5.04sec
GHF	4.6%	9.48sec
SGHF	5%	6.9sec

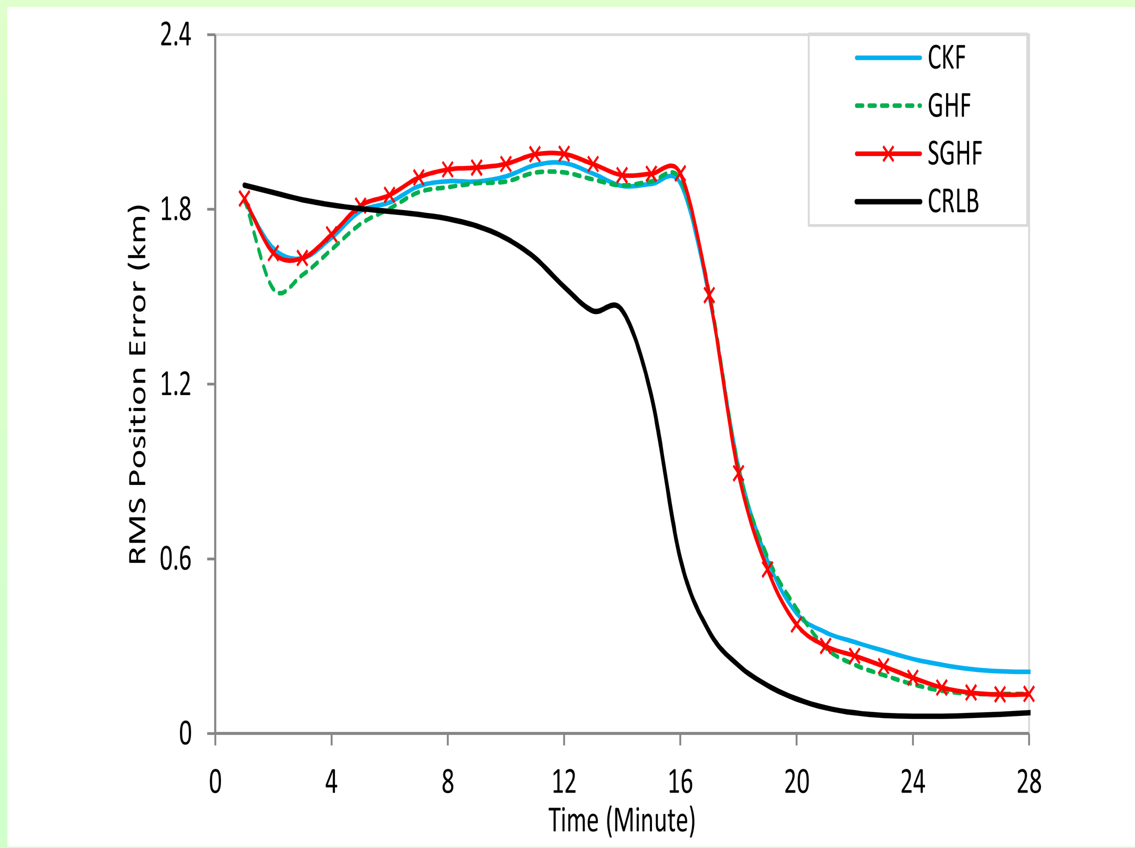


Figure 3: RMS position error of different filters

Discussions and Conlusions

- Performance of CKF, GHF and SGHF was studied and compared for a passive BOT problem.
- Better computational efficiency of SGHF when compared to GHF and high accuracy levels than CKF, makes it's performance superior to both the filters.
- Hence, SGHF can be proposed as an alternate filtering algorithm for the kind of BOT problem discussed in this work.

References

[1] B. Ristic *et al*, "Beyond the Kalman filter: Particle filters for tracking applications," *Artech house Boston*, 685, 2004.

[2] I. Arasaratnam *et al*, "Discrete-time nonlinear filtering algorithms using Gauss-Hermite quadrature," *Proc. IEEE*, 95(5), pp. 953-977, 2007.

[3] B. Jia *et al*, "Sparse-grid quadrature nonlinear filtering," *Automatica*, 48(2), pp. 327-341, 2012.