IMPROVED HIGH-DEGREE CUBATURE KALMAN FILTER

*Abhinoy Kumar Singh and Shovan Bhaumik**
Department of Electrical Engineering,
Indian Institute of Technology Patna,
Email: abhinoy@iitp.ac.in.

ABSTRACT

In this paper, an improved cubature rule based filter has been proposed for solving the nonlinear filtering problems. The intractable integral appeared while solving the nonlinear filtering problems is decomposed into the surface and the line integrals. The surface integral is solved by using the arbitrary odd degree spherical cubature rule and the line integral is solved with any order Gauss-Laguerre quadrature rule. The proposed filter is named as Improved high-degree cubature Kalman filter (IHDCKF).

INTRODUCTION

- A nonlinear system is represented as

  state model $\quad x_{k+1}=\phi(x_k)+\eta_k \quad (1)$

  measurement model $\quad y_k=\gamma(x_k)+v_k \quad (2)$

  where $x_k \in \Re^n$ and $y_k \in \Re^p$ are the state and the measurement respectively at k^{th} instant *i.e.* $k \in \{0,1,2,3,...,N\}$, $\phi(x_k)$ and $\gamma(x_k)$ are known nonlinear functions of x_k, $\eta_k \in \Re^n$ and $v_k \in \Re^p$ are white noises and normally distributed with zero mean and covariances Q_k and R_k respectively.
- In last decade, the cubature Kalman filter (CKF) [1] was most demanded due to its high accuracy at low computational load.
- In CKF, the intractable integrals are decomposed into the spherical and the line integrals. The spherical integral is approximated using 3^{rd}-degree spherical rule while the line integral is computed using first order radial rule.
- The accuracy of CKF is further enhanced in cubature quadrature Kalman filter (CQKF) [2] and the high-degree cubature Kalman filter (HDCKF) [3]. In CQKF, the line integral could be solved by using arbitrary order Gauss-Laguerre quadrature rule while the HDCKF approximates the spherical integral by using the arbitrary order spherical cubature rule.
- In the proposed Improved high-degree cubature Kalman filter (IHDCKF), the spherical integral is computed by using the arbitrary order spherical cubature rule while the line integral is approximated by using the arbitrary order Gauss-Laguerre quadrature.

INTEGRAL DECOMPOSITION

Theorem 1 *For any arbitrary function $f(X)$, $X \in \Re^n$ the integral of interest*

$$I(f)=\frac{1}{\sqrt{|\Sigma|(2\pi)^n}}\int_{\Re^n} f(X)e^{-\frac{1}{2}(X-\mu)^T\Sigma^{-1}(X-\mu)}dX$$

can be decomposed into two integrals as

$$I(f)=\frac{1}{\sqrt{(2\pi)^n}}\int_{r=0}^{\infty}\int_{U_n}[f(CrZ+\mu) \, d\sigma(Z)]r^{n-1}e^{-r^2/2}dr \quad (3)$$

where $X=CrZ+\mu$, C is Cholesky decomposition of covariance matrix Σ, $\| Z \|=1$, μ is the mean of Gaussian distribution and U_n is the surface of an unit hyper-sphere.
The proof is given in [3]. $\square$
For 0 mean and unity covariance system, the spherical integral is $\int_{U_n} f(rZ)d\sigma(Z)$. This integral is solved using arbitrary odd degree spherical cubature rule.

HIGH DEGREE CUBATURE RULE

Theorem 2 *[3] For arbitrary but odd degree spherical cubature rul, the surface integral of the form $I_{U_n}(f_{rZ})=\int_{U_n} f(rZ)d\sigma(Z)$ could be evaluated as*

$$I_{U_n,2m+1}(f_{rZ})=\sum_{|p|} w_p f\{ru_p\} \quad (4)$$

where $I_{U_n,(2m+1)}(f_{rZ})$ represents spherical integration of the function $f(rZ)$ with $(2m+1)_{th}$ degree spherical cubature rule *i.e.* $m \in Z^+$, ru_p and w_p are cubature points and corresponding weights given by

$$\{ru_p\} \triangleq \bigcup(\beta_1 ru_{p_1}, \beta_2 ru_{p_2}, \cdots, \beta_n ru_{p_n})$$

$$w_p \triangleq 2^{-n_z(u_p)}\left(I_{U_n}\left(\prod_{i=1}^{n}\prod_{j=0}^{p_i-1}\frac{z_i^2-u_j^2}{u_{p_i}^2-u_j^2}\right)\right)$$

where p is a set of non-negative numbers *i.e.* $p=[p_1,p_2,...,p_n]$, and $|p|=p_1+p_2+...+p_n$; u_p is also a set of non-negative numbers (not necessarily an integer); $n_z(u_p)$ gives the number of non-zero elements in u_p; $\beta_i=\pm 1$ and $u_{p_i}=\sqrt{p_i/m}$.
The proof is provided in [3].

GAUSS-LAGUERRE QUADRATURE RULE

Any integral of the form $\int_{\lambda=0}^{\infty} f(\lambda)\lambda^{\alpha}e^{-\lambda}d\lambda$ can be approximated using quadrature points and weights associated with them. The quadrature points can be determined from the roots of n' order Chebyshev-Laguerre polynomial

$$L_{n'}^{\alpha}(\lambda)=(-1)^{n'}\lambda^{-\alpha}e^{\lambda}\frac{d^{n'}}{d\lambda^{n'}}\lambda^{\alpha+n'}e^{-\lambda}=0$$

If the quadrature points are $\lambda_{i'}$, then the weights can be computed as $\omega_{i'}=n'!\Gamma(\alpha+n'+1)/\lambda_{i'}[\dot{L}_{n'}^{\alpha}(\lambda_{i'})]^2$ and the integral could be approximated as

$$\int_{\lambda=0}^{\infty} f(\lambda)\lambda^{\alpha}e^{-\lambda}d\lambda \approx \sum_{i'=1}^{n'}\omega_{i'}f(\lambda_{i'}) \quad (5)$$

HIGH-DEGREE CUBATURE QUADRATURE RULE

Theorem 3 *The integral $I(f)$ can be approximated as*

$$I(f)=\frac{1}{2\sqrt{\pi^n}}\sum_{i'=1}^{n'}\omega_{i'}\left[\sum_{|p|}w_p f\left\{\sqrt{2\lambda_{i'}}u_p\right\}\right]$$

Proof: Considering zero mean and unity covariance, the above integral becomes

$$I(f)=\frac{1}{\sqrt{(2\pi)^n}}\int_{r=0}^{\infty}\int_{U_n}[f(rZ)d\sigma(Z)]\, r^{n-1}e^{-\frac{r^2}{2}}dr$$

Substituting equation (4) and substituting $t=r^2/2$, we get

$$I(f)=\frac{1}{2\sqrt{\pi^n}}\int_{t=0}^{\infty}\sum_{|p|}w_p f\left\{\sqrt{2t}u_p\right\}t^{\frac{n}{2}-1}e^{-t}dt$$

After substituting $\alpha=n/2-1$, the Gauss-Laguerre quadrature rule could be applied to solve the above integral. For i' number of quadrature points denoted as $\lambda_{i'}$ the integral becomes

$$I(f)=\frac{1}{2\sqrt{\pi^n}}\sum_{i'=1}^{n'}\omega_{i'}\left[\sum_{|p|}w_p f\left\{\sqrt{2\lambda_{i'}}u_p\right\}\right]$$

$\square$

SIMULATION RESULTS

The proposed filter has been applied to estimate the states of a nonlinear system for which $\phi(x_k)=20cos(x_k)$ and $\gamma(x_k)=\sqrt{1+x_k^Tx_k}$. The initial truth states are considered as $x_0=0.1\times 0_{n\times 1}$. The filter is initialized with a value of $\hat{x}_0$ and P_0, where $\hat{x}_0=0_{n\times 1}$ and $P_0=I_n$. The error covariance matrices are given as $Q=I_n$ and $R=1$.

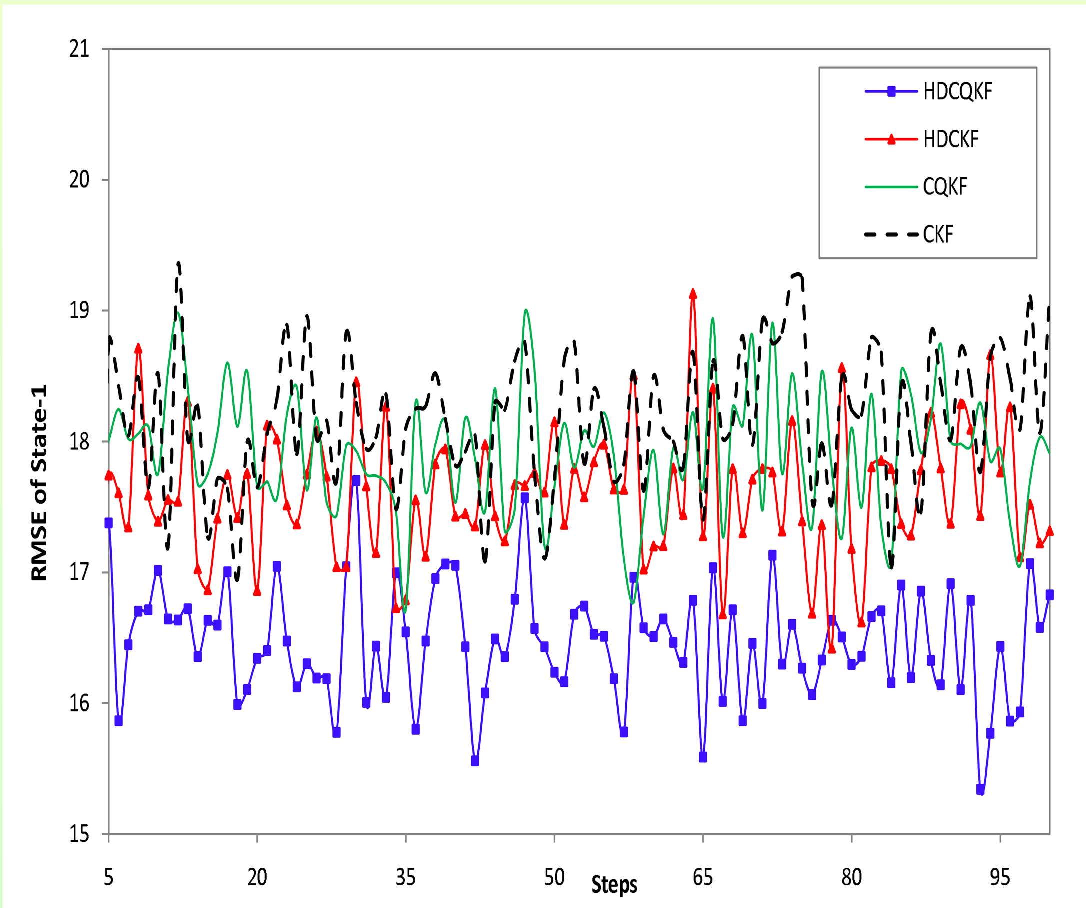

Figure 1: RMSE plot over 100 steps of 1^{st} state of a six dimesional system for 500 Monte-Carlo runs

DISCUSSIONS AND CONLUSIONS

- The quadrature filters *viz.* GHF and SGHF are applied to solve a constant turn rate maneuvering target tracking problem.
- Quadrature filters show higher accuracy than its counterparts UKF, CKF.
- SGHF could reduce the *curse of dimensionality* problem drastically, appeared with GHF [1].

SELECTED REFERENCES

[1] I. Arasaratnam and S. Haykin, “Cubature Kalman filter,” *IEEE Trans. Auto. Control*, 54 (6), Jun. 2009, pp. 1254-1269.
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